Centroids of Center of Gravity

$$W . \bar{x} = \int x \, dw \rightarrow V . \bar{x} = \int x \, dV \rightarrow A \bar{x} \rightarrow L \bar{x}$$

$$W . \bar{y} = \int y \, dw \rightarrow V . \bar{y} = \int y \, dv \rightarrow A \bar{y} \rightarrow L \bar{y}$$

$$W . \bar{z} = \int z \, dw \rightarrow V . \bar{z} = \int z \, dv \rightarrow A \bar{z} \rightarrow L \bar{z}$$

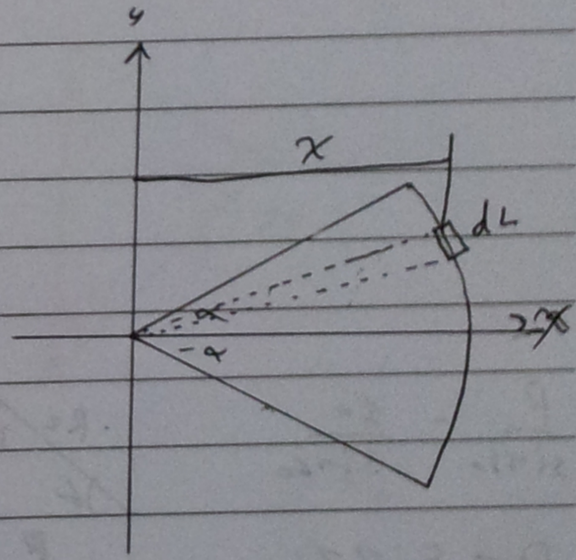
$$L = \int_{-\infty}^{\alpha} r \, d\theta$$

$$= 2 \int_{0}^{\alpha} r \, d\theta$$

$$= 2r \int_{0}^{\alpha} d\theta$$

$$= 2r \theta \Big]_{0}^{\alpha}$$

$$= 2 \alpha r$$



$$L * \bar{x} = \int_{-\alpha}^{\alpha} x \, dl$$

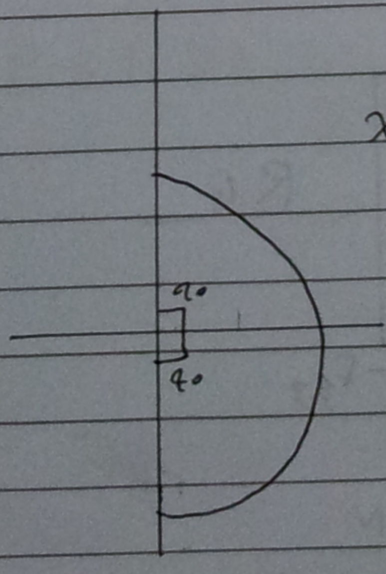
$$2\alpha r \, \bar{x} = \int_{-\alpha}^{\alpha} r \cos\theta \, r \, d\theta$$

$$2\alpha r \, \bar{x} = 2r^2 \int_{0}^{\alpha} \cos\theta \, d\theta$$

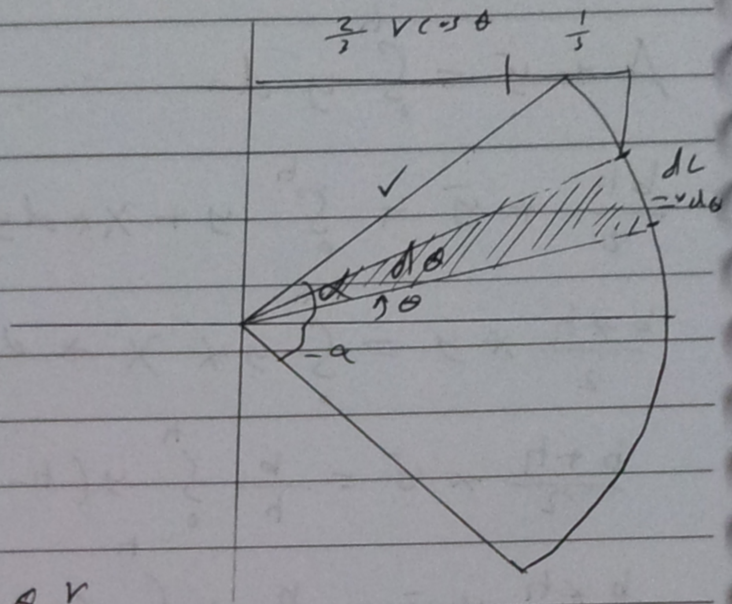
$$\alpha \bar{x} = r \sin\alpha$$

$$\boxed{\bar{x} = \frac{r \sin\alpha}{\alpha}}$$

$$\bar{x} = \frac{2r}{\pi}$$



$$A \cdot \bar{x} = \int_{-b}^{\alpha} x \, da$$



$$A = \int_{-b}^{\alpha} da$$

$$= 2 \int_{0}^{\alpha} \frac{\overset{dL}{\boxed{r\, d\theta}} * r}{2}$$

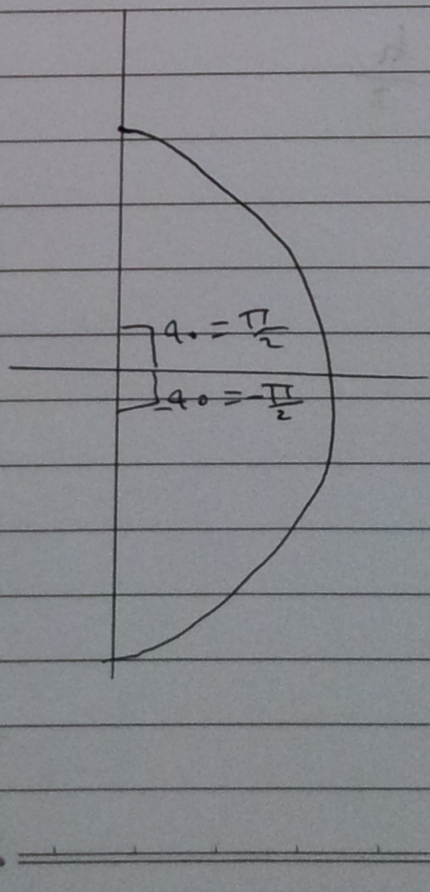
$$A = r^2 * \alpha$$

$$r^2 \alpha \bar{x} = \int_{-\alpha}^{\alpha} \frac{2}{3} r \cos\theta * r\, d\theta \frac{r}{2}$$

$$r^2 \alpha * \bar{x} = \cancel{2} * \frac{2}{3} * \frac{r^3}{\cancel{2}} \int_{0}^{\alpha} \cos\theta \, d\theta$$

$$\bar{x} = \frac{2}{3} r \frac{\sin\alpha}{\alpha}$$

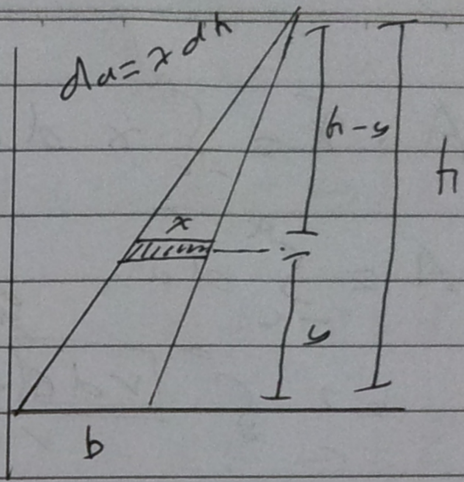
$$\bar{x} = \frac{2}{3} r \frac{\sin\frac{\pi}{2}}{\frac{\pi}{2}}$$



$$\bar{x} = \frac{4}{3} \frac{1}{\pi} r$$

$$\bar{x} = \frac{4}{3\pi} r$$

$$\bar{x} = 0.424\, r$$



$$A * \bar{y} = \int_0^h y\, dA$$

$$\frac{bh}{2} * \bar{y} = \int_0^h y * x * dy$$

$$\frac{b*h}{2} * \bar{y} = \int_0^h y * x * dy$$

$$\frac{b*h}{2} * \bar{y} = \frac{b}{h} \int_0^h y(h-y)\, dy$$

$$\frac{x}{h-y} = \frac{b}{h}$$

$$\frac{b*h}{2} * \bar{y} = \frac{b}{h} * \int_0^h (hy - y^2)\, dy$$

$$\frac{bh}{2} * \bar{y} = \frac{b}{h} * \left[\frac{hy^2}{2} - \frac{y^3}{3}\right]_0^h$$

$$x = \frac{b}{h}(h-y)$$

$$\frac{bh}{2} * \bar{y} = \frac{b}{h} * \frac{h^3}{6}$$

$$\therefore \bar{y} = \frac{h}{3}$$