Q26) P.464)

$$\int_{\ln\frac{\pi}{6}}^{\ln\frac{\pi}{2}} 2e^{x}\cos(e^{x})\,dx$$

Sol) let $u = e^{x}$

$du = e^{x}dx$ when $x = \ln\frac{\pi}{6}$, $u = \frac{\pi}{6}$

$x = \ln\frac{\pi}{2}$ $u = \frac{\pi}{2}$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 2\cos u\,du = \left[2\sin u\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

$$= 2\left[\sin\frac{\pi}{2} - \sin\frac{\pi}{6}\right]$$

$$= 2\left[1 - \tfrac{1}{2}\right] = 1$$

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O the Exponential and Logarithmic functions

The Function a^{x}. If a is a positive number and x is any number. then the function.

$$\left[\, a^{x} = e^{x\ln a} \;,\; a > 0 \,\right]$$

$$\left[\, e^{x\ln a} = e^{\ln a^{x}} = a^{x} \,\right]$$

Ex) $2^{\sqrt{3}} = e^{\sqrt{3}\ln 2}$ Ex) $3^{\pi} = e^{\pi \ln 3}$

Laws of a^x:

IF $a>0$ and any x & y.

1) $a^{x+y} = a^x \cdot a^y$, 2) $a^{-x} = \frac{1}{a^x}$

3) $a^{x-y} = \frac{a^x}{a^y}$, 4) $(a^x)^y = a^{xy} = (a^y)^x$

The derivative of $y=a^x$:

$$\frac{d}{dx}(a^x) = \frac{d}{dx}(e^{x\ln a}) = e^{x\ln a} \cdot \frac{d}{dx}(x\ln a)$$

$$= e^{x\ln a} \cdot \ln a$$

$$\frac{d}{dx} a^x = a^x \ln a$$

Ex) $\frac{d}{dx} 3^x = 3^x \cdot \ln 3$

* IF $a>0$ and u is a differentiable func of x

then a^u is differentiable func. of x,

$$\boxed{\frac{d}{dx}(a^u) = a^u \cdot \ln a \cdot \frac{du}{dx}}$$

Ex) $\frac{d}{dx} 5^{\sin x} = 5^{\sin x} \cdot \ln 5 \cdot \cos x$

Ex) $\frac{d}{dx} 7^{\tan x} = 7^{\tan x} \ln 7 \sec^2 x$

by logarithmic differentia..

$y = 7^{\tan x}$

$\ln y = \tan x \ln 7$

$\frac{1}{y} \frac{dy}{dx} = \sec^2 x \ln 7$

$\therefore \frac{dy}{dx} = y \sec^2 x \ln 7$

$\frac{dy}{dx} = 7^{\tan x} \sec^2 x \ln 7$

$\boxed{\int a^u \, du = \frac{a^u}{\ln a} + c}$, provided

IF $\ln a \neq 0$

$\therefore a \neq 1$

Ex) $\int 2^x dx = \frac{2^x}{\ln 2} + c$

$\int 2^{\sin x} \cos x \, dx = \frac{2^{\sin x}}{\ln 2} + c$

$\int 3^{\tan x} \ln 3 \sec^2 x \, dx = 3^{\tan x} + c$

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The graph of $y=a^x$:

$$\frac{da}{dx} = a^x \ln a \qquad a>0$$

$= +$ IF $a>1$ (rising)

$= -$ IF $0<a<1$ (falling)

$$\frac{d^2a}{dx^2} = a^x (\ln a)^2$$

$= +$ (The curve concave up)

IF $y=a^x$ $a=2$

$y=2^x$

I $a=\frac{1}{2}$

$y=2^{-x}$