Chapter Two

Force vectors

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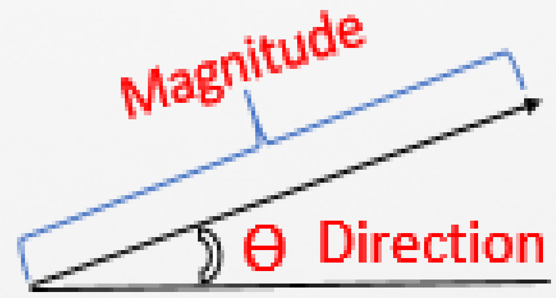
Scalars and Vectors

Scalar (A)

A scalar is any positive or negative physical quantity that can be completely specified by its **magnitude** for example: length, mass, and time.



Vector ($\vec{A}$)

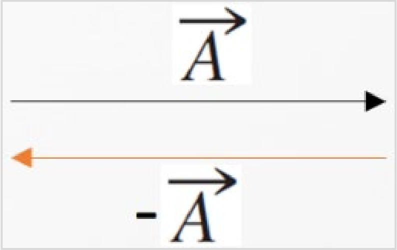
A vector is any physical quantity that requires both a magnitude and a direction for its complete description for example: force, position, and moment.

- The length of the arrow represents the *magnitude* of the vector.
- The angle θ between the vector and a fixed axis defines the *direction of its line of action*.
- The head or tip of the arrow indicates the *sense of direction* of the vector.

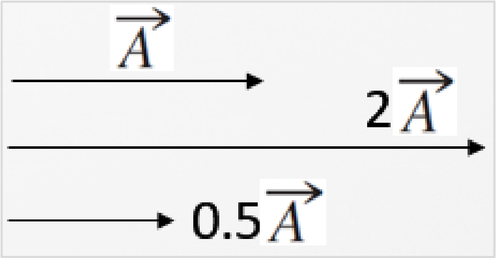
Vector Operations

- **Direction**

The negative sign means the opposite direction.



- **Multiplication and Division of a Vector by a Scalar**
  - If a vector is multiplied by a positive scalar, its magnitude is increased by that amount.
  - Multiplying by a negative scalar will also change the directional sense of the vector.



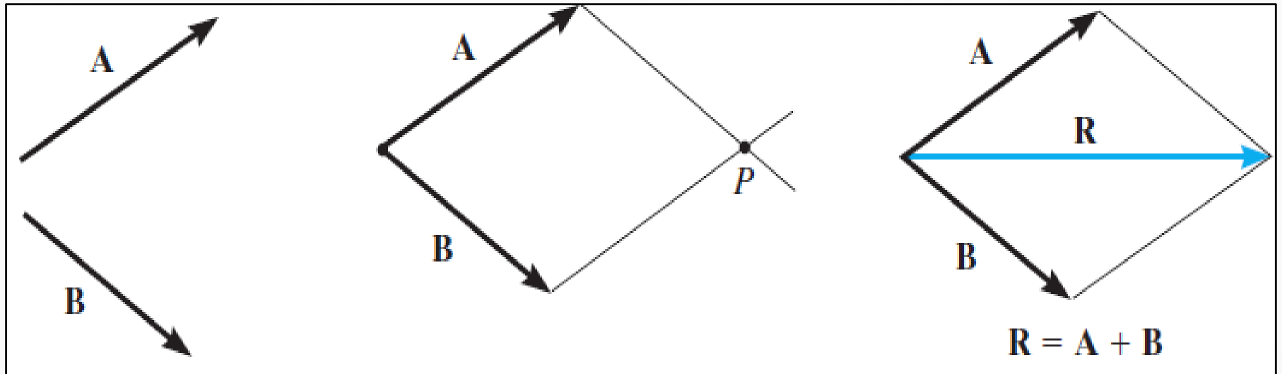
- **Vector Addition**

There are two approaches:

1- Parallelogram law of addition.

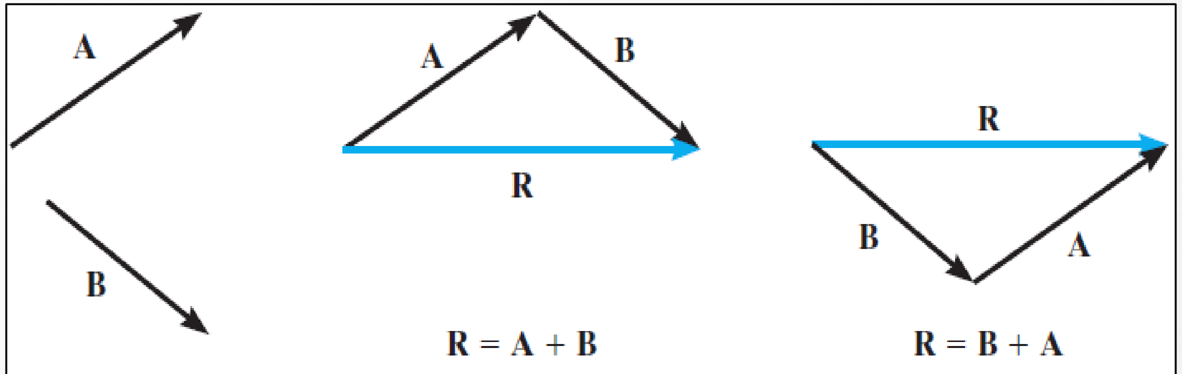
2- Triangle rule.

1- Parallelogram law of addition



Parallelogram law

2- Triangle rule

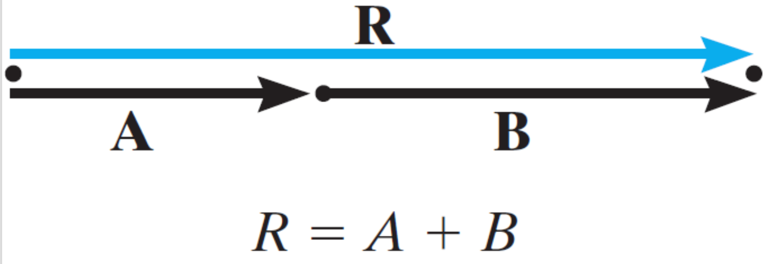


Triangle rule

Triangle rule

As a special case:

If the two vectors A and B are collinear , i.e., both have the same line of action, the parallelogram law reduces to an algebraic or scalar addition R = A + B , as shown below:



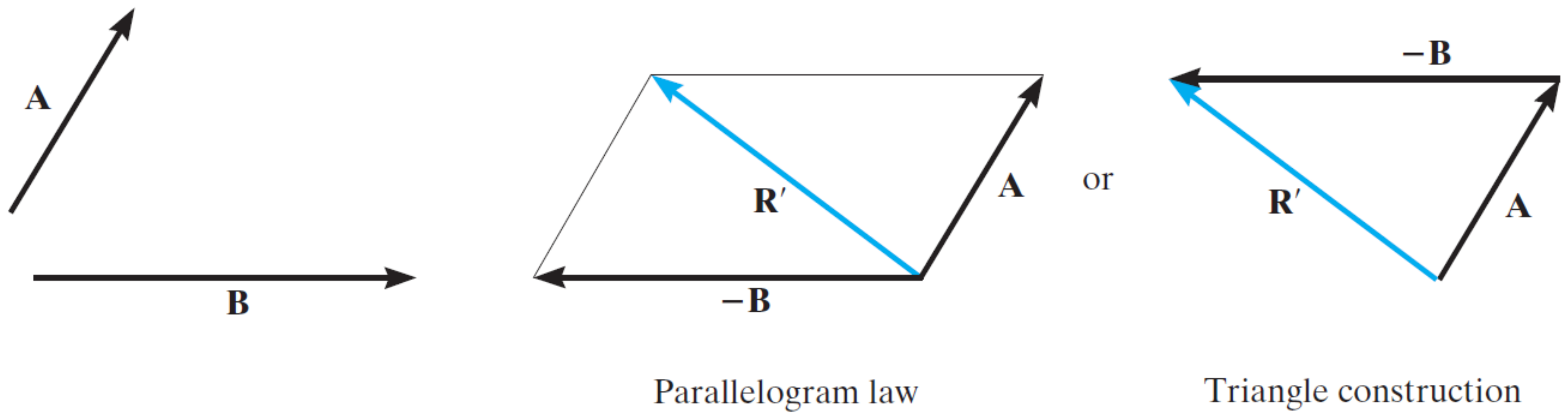
Addition of collinear vectors

Vector Subtraction

The resultant of the difference between two vectors A and B of the same type may be expressed as:

$$R' = A - B = A + (-B)$$

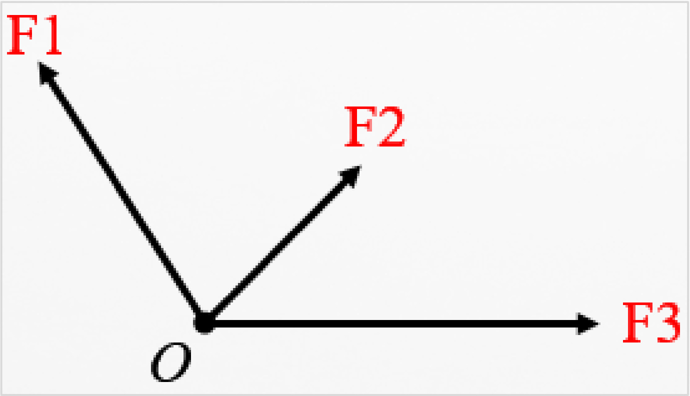
Subtraction is therefore defined as a special case of addition, so the rules of vector addition also apply to vector subtraction.



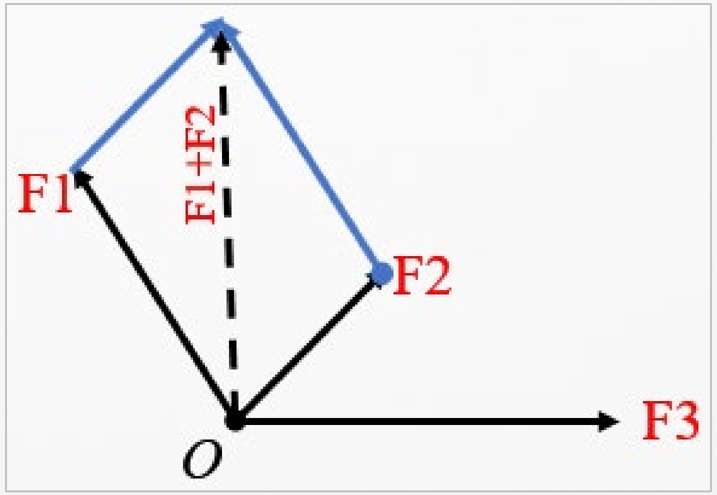
Vector subtraction

Addition of Several Forces

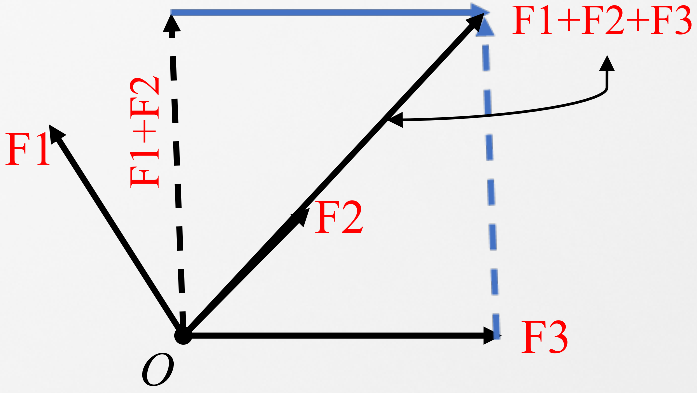
If more than two forces are to be added, successive applications of the parallelogram law can be carried out in order to obtain the resultant force. For example, if three forces **F1** , **F2** , **F3** act at a point *O*.



(1)



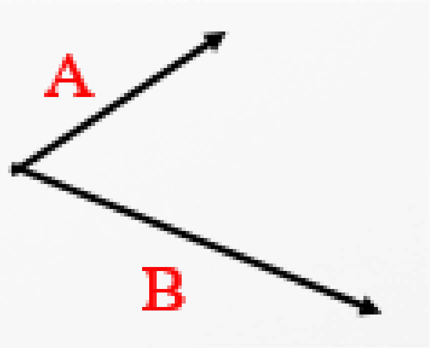
(2)

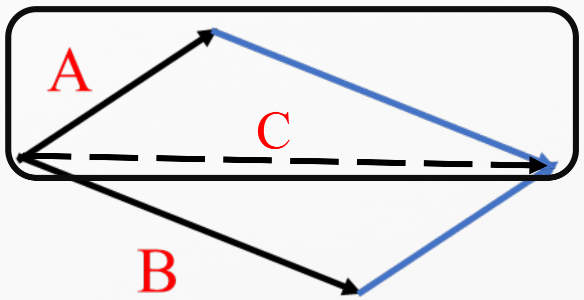


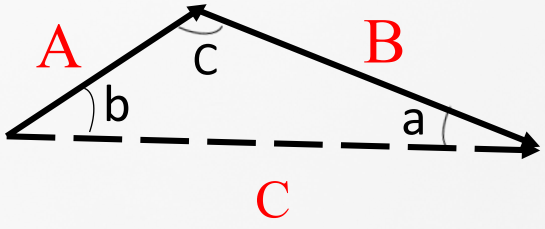
(3)

Trigonometry

we can utilize trigonometry to find resultants force:







From Cosine Law: $C = \sqrt{A^2 + B^2 - 2AB\cos(c)}$

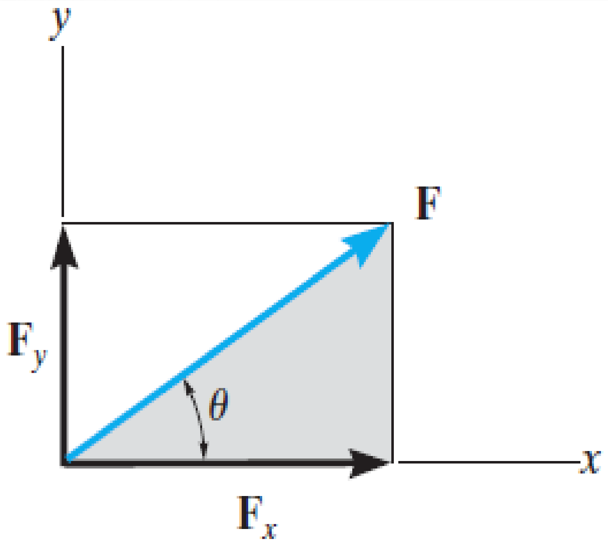
From Sines Law: $\dfrac{A}{\sin a} = \dfrac{B}{\sin b} = \dfrac{C}{\sin c}$

Addition of a System of Coplanar Forces

When a force is resolved into two components along the *x* and *y* axes, the components are then called *rectangular components*.

Notation of Rectangular Components

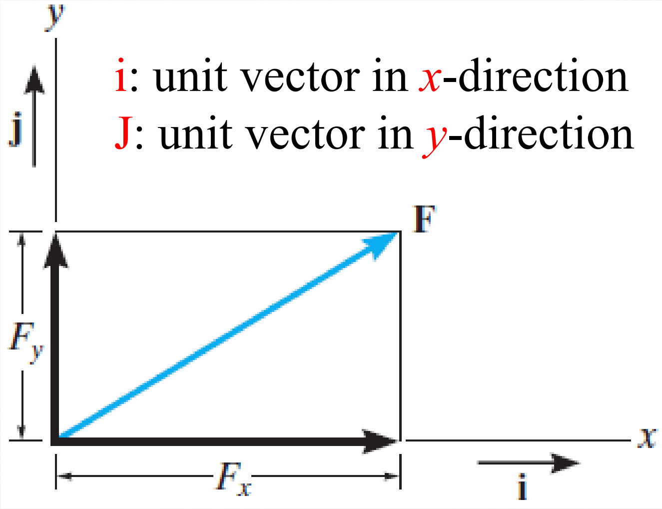
Scalar Notation



$$Fx = F\cos\Theta$$
$$Fy = F\sin\Theta$$
$$F = \sqrt{(Fx)^2 + (Fy)^2}$$
$$\Theta = \tan^{-1}\frac{Fy}{Fx}$$

Fx and *Fy* are vector componentes

Cartesian Vector Notation



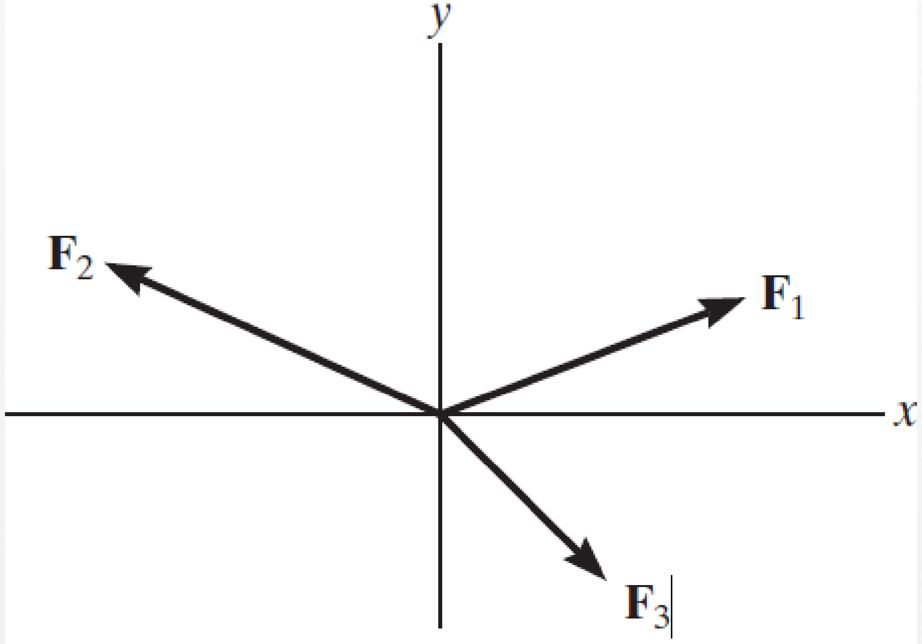
$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j}$$

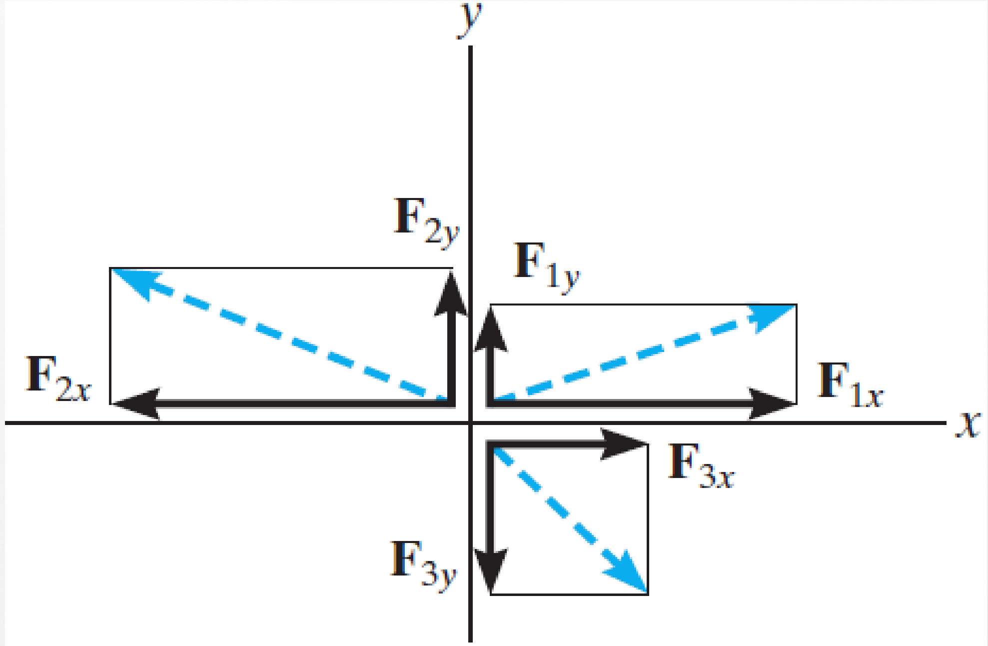
Coplanar Force Resultants

We can use either of the two methods just described to determine the resultant of several *coplanar forces*.

To do this:

- Each force is first resolved into its x and y components
- And then the respective components are added using scalar algebra since they are collinear.
- The resultant force is then formed by adding the resultant components using the parallelogram law.





1- Each force is first represented as a Cartesian vector, i.e.,

$$\mathbf{F}_1 = F_{1x}\mathbf{i} + F_{1y}\mathbf{j}$$
$$\mathbf{F}_2 = -F_{2x}\mathbf{i} + F_{2y}\mathbf{j}$$
$$\mathbf{F}_3 = F_{3x}\mathbf{i} - F_{3y}\mathbf{j}$$

2- Add the same components together (x-components, y-components) and the vector resultant (F_R) results from three forces (F_1, F_2, F_3).

$$\begin{aligned}\mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 \\ &= F_{1x}\mathbf{i} + F_{1y}\mathbf{j} - F_{2x}\mathbf{i} + F_{2y}\mathbf{j} + F_{3x}\mathbf{i} - F_{3y}\mathbf{j} \\ &= (F_{1x} - F_{2x} + F_{3x})\mathbf{i} + (F_{1y} + F_{2y} - F_{3y})\mathbf{j} \\ &= (F_{Rx})\mathbf{i} + (F_{Ry})\mathbf{j}\end{aligned}$$

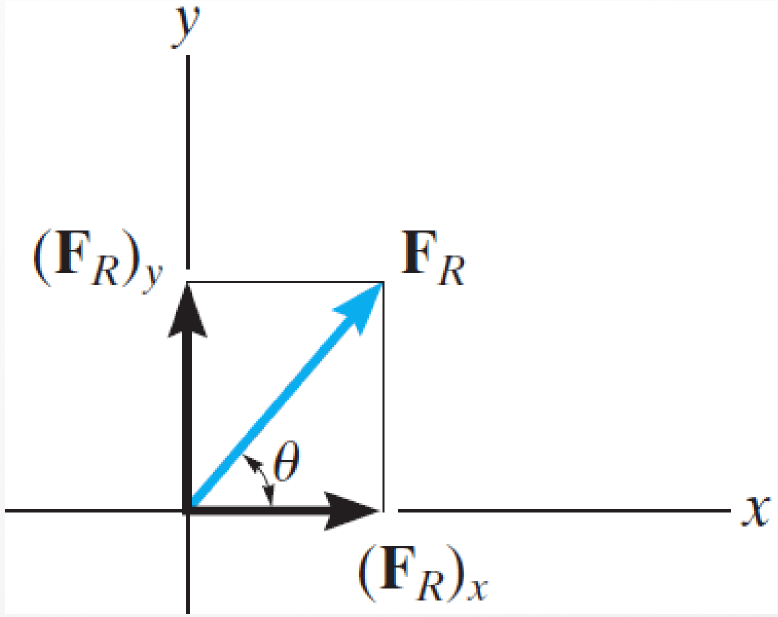
When F_{Rx} represents as forces resultant in x-direction

F_{Ry} represents as forces resultant in y-direction

3- By using F_{Rx} and F_{Ry}, we can apply *Parallelogram law to find the resultant* (F_R).

If scalar notation is used, we have:

$$(\xrightarrow{+}) \qquad (F_R)_x = F_{1x} - F_{2x} + F_{3x}$$
$$(+\uparrow) \qquad (F_R)_y = F_{1y} + F_{2y} - F_{3y}$$



So,

$$\boxed{\begin{aligned}(F_R)_x &= \Sigma F_x \\ (F_R)_y &= \Sigma F_y\end{aligned}}$$

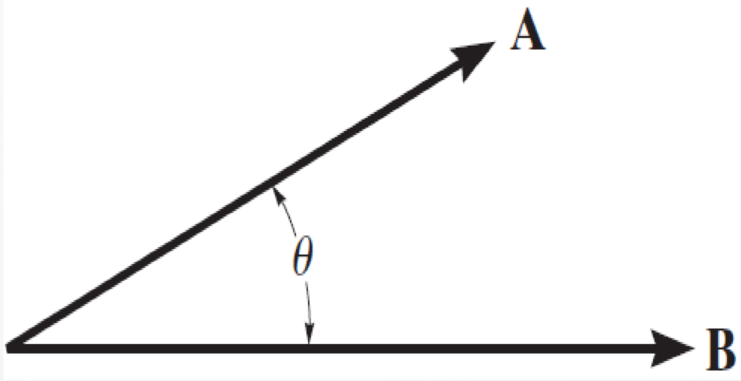
$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2}$$

$$\theta = \tan^{-1}\left|\frac{(F_R)_y}{(F_R)_x}\right|$$

Dot Product

The dot product, which defines a particular method for "multiplying" two vectors, is used to find the angle between two lines or the components of a force parallel and perpendicular to a line.

The *dot product* of vectors **A** and **B** , written **A** · **B** , and read " **A** dot **B** " is defined as the product of the magnitudes of **A** and **B** and the cosine of the angle *Ө* between them, expressed in equation form:



$$\boxed{\mathbf{A} \cdot \mathbf{B} = AB\cos\theta}$$

Laws of Operation.

1. Commutative law: A . B = B . A

2. Multiplication by a scalar: a(A . B) = (aA) . B = A . (aB)

3. Distributive law: A . (B + D) = (A . B) + (A . D)

Cartesian Vector Formulation

If we want to find the dot product of two general vectors **A** and **B** that are expressed in Cartesian vector form, then we have:

$$\hat{\mathbf{i}}\cdot\hat{\mathbf{i}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{k}} = 1$$

$$\hat{\mathbf{i}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{i}} = 0$$

$$\hat{\mathbf{i}}\cdot\hat{\mathbf{k}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{i}} = 0$$

$$\hat{\mathbf{j}}\cdot\hat{\mathbf{k}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{j}} = 0$$

$$\begin{aligned}\mathbf{A}\cdot\mathbf{B} &= (A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k})\cdot(B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}) \\ &= A_xB_x(\mathbf{i}\cdot\mathbf{i}) + A_xB_y(\mathbf{i}\cdot\mathbf{j}) + A_xB_z(\mathbf{i}\cdot\mathbf{k}) \\ &\quad + A_yB_x(\mathbf{j}\cdot\mathbf{i}) + (A_yB_y(\mathbf{j}\cdot\mathbf{j}) + A_yB_z(\mathbf{j}\cdot\mathbf{k}) \\ &\quad + A_zB_x(\mathbf{k}\cdot\mathbf{i}) + A_zB_y(\mathbf{k}\cdot\mathbf{j}) + A_zB_z(\mathbf{k}\cdot\mathbf{k})\end{aligned}$$

Carrying out the dot-product operations, the final result becomes

$$\boxed{\mathbf{A}\cdot\mathbf{B} = A_xB_x + A_yB_y + A_zB_z}$$

Problems: Chapter Two / Hibbeler's book

2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.8, 2.9, 2.10, 2.19, 2.20, 2.21, 2.23,

2.26, 2.28, 2.29, 2.30, 2.33, 2.34, 2.35, 2.37, 2.40, 2.42, 2.43, 2.44,

2.46, 2.48, 2.49, 2.50, 2.51, 2.54, 2.56, 2.57, 2.58, 2.59,