ميكانيك
مرحلة اولى G1

قسم الفيزياء

CH. 7

D

المستنصرية
ملونة

chapter 7.

Work and Energy

We have study before from Newton's second law of motion we can find the acc^n $\vec{a}$ of a particle:

$\vec{F} = m\vec{a}$ ---- Newton's 2nd law

$\therefore \vec{a} = \frac{\vec{F}}{m}$

if F & m are constants
Then $\vec{a}$ is constant, let be along the x-direction
So we can then find the speed of the particle and its position from Equ^n before:

① ---- $v = v_0 + at$ & $x = v_0 t + \frac{1}{2} a t^2$

شبعاً الازاحة هنا هي داله للزمن $x(t)$ او موقع الجسيمة هي داله للزمن

$x(t) \Rightarrow$ the position of a particle is as a function of time

when the force $\vec{F}$ acting on a particle is not constant
so the we can not use the equ^n before ①
for finding position and speed as a function of time because the acc^n $\vec{a}$ now is not cons
So to solve such problems we use the mathmatical process of integration.

هنا لنا مسائل لها شرحه لاحقاً ;

Work done by a Constant Force :-

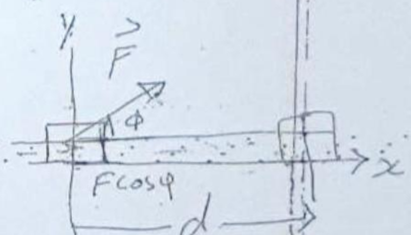
Consider a particle محرك acted on by a force (const. force) and the motion in a straight line in the direction of the force. So the work done by this force on the particle can be define as : the product of the magnitude of the force (F) and the distance (d) through which the particle moves.

$$W = F.d$$

If the force acting on a particle is not in the direction in which the particle moves, In this case we define the work done ~~by~~ as the product of the component of the force along the line of motion by the distance d the body moves along that line. تحرك

for example in the figure below:

a const. force $\vec{F}$ makes an angle ϕ with the x- axis and acts on a particle whose displacement along the x-axis is d. Then



$$W = (F\cos\phi)\,d \quad \text{——} (1)$$

Eqn-(1) refers only to the work done on the particle by the particular force $\vec{F}$.

But The worke done by the other forces (like its weight, and the frictional force exerted by the plane) must be calculated separately. The total work done on the particle is the sum of the works done by the separate forces.

– when ϕ is zero

$$W = (F\cos\phi)d = (F\cos 0)d = Fd$$

– when ϕ is 90° the force has no component in the direction of motion.

لا يوجد مركبة (القوة) باتجاه الحركة ولهذا لا تنجز شغل على الجسم من هذه القوة

امثلة على هذا النوع من القوى التي لا تنجز شغلاً هي: $\therefore W = \text{zero}$

for example ① vertical force holding a body a fixed distance off the ground. القوة العمودية المرفوعة على جسم تمنع حركته.

② the body is moved horizontally over the ground.
جسم يتحرك افقياً على الارض

③ the centripetal force acting on a body because the force is always at right angles to the direction ~~of~~ in which the body is moving. القوة المركزية لانها عمودية على اتجاه الحركة

④ body that does not move, for its displacement is then zero.

ممكن ملاحظة (Fig 7-2) صفحة (133) يوضح النقاط اعلاه.

Work is a scalar: equ. (1) shows that work is such quantity. we can write this equ. as:
عددي

$$W = \vec{F} \cdot \vec{d} \quad \text{scalar product}$$

like equ. before

$$\vec{a} \cdot \vec{b} = ab\cos\theta$$

- work can be either positive or negative.

W -ive when the component force that act on a ~~particle of motion opposite to the direction~~ force or component of force act opposite to the direction of motion, and $\phi = 180°$

- unit of work is the work done by a unit force in moving a body a unit distance in the direction of the force.

⇒ In mks system the unit of work is newton-met called 1 Joule

In British engineering system = = is pound - foot

In cgs 1 dyne - cm ⇒ 1 erg

$$\boxed{W \Longrightarrow nt.m \text{ or } dyne.cm \text{ or } ft\text{-}lb}$$

$$1 \text{ Joule} = 10^7 \text{ ergs} = 0.7376 \text{ ft-lb}$$

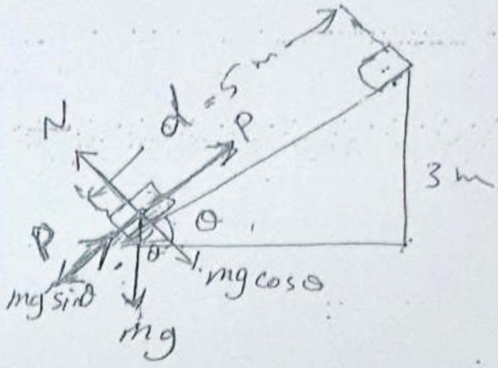
Example 1

A block of mass 10.0 kg is to be raised from يرتفع the bottom to the top of an incline 5.00 m long and ميل 3.0 m off the ground at the top. Assuming frictionless عديم الاحتكاك surfaces, how much work must be done by a force parallel to the incline pushing the block up at cons. speed ∴ a place where $g = 9.8\ m/sec^2$

$a = 0$

$\sum F_x = ma$

$\sum F_x = 0$



$W = P \cdot d$

$= Pd\cos\phi$

$\Sigma F_x = 0$

بداية يجب انه نجد قيمة القوة P
التي هي موازية للسطح المائل

$P - mg\sin\theta = 0$

$P = mg\sin\theta$

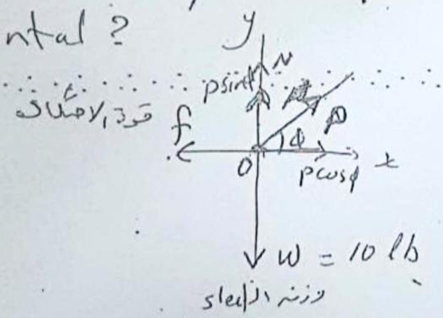
$= 10 \times 9.8 \times \frac{3}{5} = 58.8 \text{ nt}$

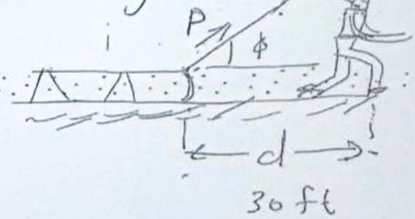
$\phi = 0$

$\therefore W = P \cdot d = Pd\cos 0 = 58.8 \times 5 = 294 \text{ J}$

Example 2

A boy pulls a 10-lb sled (زلاجة) 30 ft along a horizontal surface at a constant speed. What work does he do on the sled if the coefficient of kinetic friction is 0.2 and his pull makes an angle of 45° with the horizontal?





$W = P \cdot d = Pd\cos\phi$

$\Sigma F_x = 0$ because of cons. speed

$P\cos\phi - f = 0 \quad \text{----} \quad (1)$

$\Sigma F_y = 0$

$p\sin\phi + N - w = 0$ --- (2) $\Rightarrow N = W - p\sin\phi$

$f = \mu_k N$ ---- (3)

from (1) & (3)

$p\cos\phi - \mu_k N = 0$ ---(4)

from (2)

$N = W - p\sin\phi$ --- 2'

from (2)' & (4) we will get

$p\cos\phi - \mu_k (W - p\sin\phi) = 0$

$p\cos\phi - \mu_k W + p\mu_k \sin\phi = 0$

$p(\cos\phi + \mu_k \sin\phi) = \mu_k W$

$\therefore p = \dfrac{\mu W}{\cos\phi + \mu\sin\theta} = \dfrac{0.2 \times 10}{\frac{1}{\sqrt{2}} + 0.2/\sqrt{2}} = \dfrac{2}{0.707 + 0.144}$

$= 2.4$ lb

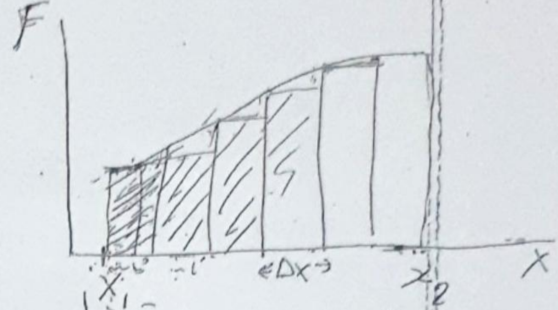
$\therefore \dot{W} = pd\cos\phi = 2.4 \times 30 \times 0.707$

$= 51$ ft-lb.

Work Done by a variable Force – one Dimensional case

Now we consider the worke done by a force that varies in magnitude only

Let F acts in the x-direction and be as a function of position $F(x)$.



القوة تقريباً ثابتة في x_1 الى $x_1+\Delta x$
ولهذا الشغل سيكون

$$\Delta W = F\,\Delta x$$

القوة في كل Δx تكون صغيرة اي غير ثابتة لكن القوة تقريباً ثابتة خلال Δx الواحد
فلايجاد الشغل الكلي في x_1 الى x_2

To find total work done by force in displacing body from x_1 to x_2, W_{12}

$$W_{12} = \sum_{x_1}^{x_2} F\,\Delta x$$

$$W_{12} = \lim_{\Delta x \to 0} \sum_{x_1}^{x_2} F\,\Delta x$$

$$W_{12} = \int_{x_1}^{x_2} F\,dx \quad \text{OR} \quad \boxed{W_{12} = \int_{x_1}^{x_2} F(x)\,dx}$$

مثال على هذا النوع من القوة المتغيرة المقدار هو السبرنك

Consider a spring attached to a wall.

If we stretch a spring to a position (x) from the origin O then:

امتداد (38)

(8)

stretch = elongate

$F = -kx$ → force law for spring

where :

F = force exerted by the spring القوة التي يسلطها

k = force constant ⟹ or spring const.

$F' = kx$

F' = force exerted on the spring القوة المسلطة

$k = \frac{F}{x} = \frac{\text{Newton}}{\text{meter}}$

the worke done $w = \int_0^x F(x)\,dx$

$= \int_0^x (kx)\,dx$

for spring → $\boxed{w = \frac{1}{2}kx^2}$

الشغل لإطالة السبرنك = الشغل لكبس السبرنك

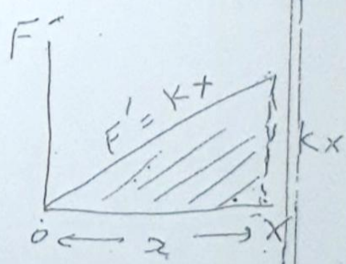
work to compress a spring by x = work to stretch it by x

لأن x^2

هناك طريقة أخرى لحساب الشغل من الرسم وذلك نبدل

التكامل نقوم بحساب المساحة تحت الرسم بين القوة والإزاحة

والإزاحة في $x=0$ الى $x=x$



Area of triangle = work done مساحة المثلث

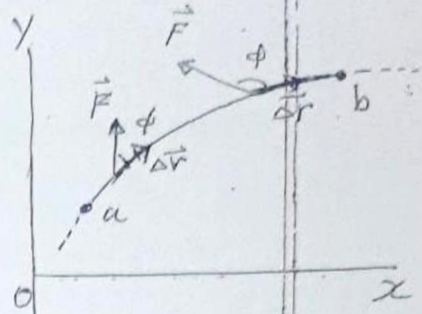
$\frac{1}{2}$ القاعدة × الارتفاع

$\frac{1}{2} x (kx) = \frac{1}{2} k x^2$

~~the~~ $\boxed{w = \frac{1}{2} k \cdot x^2}$ The same as before.

Work Done by a Variable Force – Two Dimensional case

F acting on a particle may vary in direction as well as in magnitude and the particle move along a curved path



$$dW = \vec{F}\cdot\Delta\vec{r}$$
$$= F\cos\phi\,\Delta r$$

On line from $a \rightarrow b$

on the limit $\Delta r \rightarrow 0$

$$W_{ab} = \int_a^b \vec{F}\cdot d\vec{r} = \int_a^b F\cos\phi\,dr$$

OR

$$\vec{F} = \hat{i}F_x + \hat{j}F_y$$
$$d\vec{r} = \hat{i}dx + \hat{j}dy$$

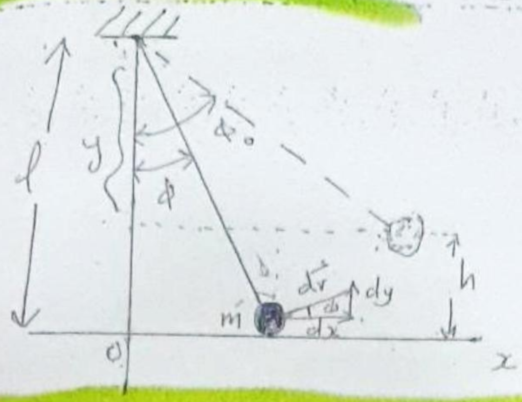
$$\left\{ \begin{array}{l} \hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = 1 \\ \hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{i} = 0 \end{array} \right\}$$

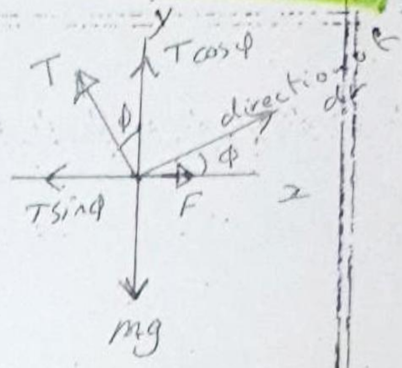
$$\vec{F}\cdot d\vec{r} = F_x dx + F_y dy$$

$$\therefore W_{ab} = \int_a^b (F_x\,dx + F_y\,dy)$$

is called line integral.

Example 3 : particle of mass m suspended معلقة from a weightless cord خيط of length l. This is called a simple pendulum. Let us displace the particle along a circular path of radius l from $\phi = 0$ to $\phi = \phi_0$ by applying a force that always horizontal. مثال للقوة المتغيرة





we can apply such a force by pulling the particle
will then displaced a vertical distance h.
(no acceleration), $T \Rightarrow$ tension in the cord
$\Sigma F = 0$ $\quad F \Rightarrow$ applied force
$mg \Rightarrow$ weight of particle

work done as the mass m moves from $\phi = 0$ to $\phi = \phi_0$
under the action of the force $\vec{F}$ is

$$W = \int_{\phi=0}^{\phi=\phi_0} \vec{F} \cdot d\vec{r} = \int_{\phi=0}^{\phi=\phi_0} F \cos\phi \, dr$$

$F_x - T\sin\phi = 0$

$F_x = T\sin\phi$ --- (1)

$mg = T\cos\phi$ -- (2)

divided (1) by (2)

$F_x = mg\tan\phi$

$\tan\phi_0 = \frac{x}{l-h}$; $x = (l-h)\tan\phi_0$

when $\phi = 0$, $x = 0$; $y = 0$

when $\phi = \phi_0$ $x = (l-h)\tan\phi_0$, $y = h$

$$\therefore w = \int_{x=0,\,y=0}^{x=(l-h)\tan\phi_0,\; y=h} (F_x\, dx + F_y\, dy)$$

$F_y = 0$ ($\because f$ is long x-axis) لا توجد قوة على y

$$\therefore W = \int_{x=0,\,y=0}^{x=(l-h)\tan\phi_0,\; y=h} mg \tan\phi\, dx$$

$\tan\phi = \frac{dy}{dx} \Rightarrow$ $dy = \tan\phi\, dx$

$$\therefore W = \int_{y=0}^{y=h} mg\, dy = [mgy]_0^h = mgh$$

$\boxed{\therefore W = mgh}$ ✓ Work W is energy transferred to ($+ve$ W) or from ($-ve$ W) an object by means of a force acting on it

Kinetic Energy and Work-Energy Theorem

نظرية الطاقة الحركية وطاقة الشغل

$\Sigma F = 0$ before we study unacc^n objects بشكل

But if $\Sigma F \neq 0$

$\therefore$ the object is accelerated الجسم معجل

consider a constant force $\vec{F}$ acting on a particle of mass (m) it will produce a constant acc^n $\vec{a}$

$a = \frac{v - v_0}{t}$, $x = \frac{v + v_0}{2} t$

$v \to$ speed at time t
$v_0 \to$ " " " $t = 0$

$w = Fx = max$

(12)

$$= m\left(\frac{v-v_0}{t}\right)\left(\frac{v+v_0}{2}\right)t = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

$$W = \frac{1}{2}m\left(v^2 - v_0^2\right) \quad \leftarrow$$ {holds for constant or variable force}

$$K = \frac{1}{2}mv^2 \quad , \quad K_0 = \frac{1}{2}mv_0^2$$

where $K \longrightarrow$ kinetic energy of the body.

$W = K - K_0 \longrightarrow$ this eqn called work-energy theorem for a particle

$W = \Delta K$

work can be (+ve) or (−ve)

{ worke done equal to the change in kinetic energy. i.e when K decreas W(−ve)

For the force vary in magnitude (but not in direction) i.e the displacement to be in the direction of the force let be x-axis, the work done by resultant force in displacing the particle from x_0 to x is

$$W = \int \vec{F} \cdot d\vec{r} = \int_{x_0}^{x} F\, dx$$

from Newton's second law: $F = ma$ and $a =$

$$a = \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt} = \frac{dv}{dx} v = v\frac{dv}{dx}$$

Hence $$W = \int_{x_0}^{x} F\, dx = \int_{x_0}^{x} mv \frac{dv}{dx}\, dx = \int_{v_0}^{v} mv\, dv = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

which is same as before (1)

Example 4// A neutron one of the constituents of a nucleus, is found to pass two points 6.0 m apart in a time interval of 1.8×10^{-4} sec. Assuming its speed was constant, find its kinetic energy. The mass of a neutron is 1.7×10^{-27} kg.

The speed is obtained from

$$v = \frac{d}{t} = \frac{6.0 \text{ meter}}{1.8 \times 10^{-4} \text{ sec}} = 3.3 \times 10^{4} \text{ m/sec}$$

$$K = \frac{1}{2} m v^2 = \frac{1}{2} \times 1.7 \times 10^{-27} \times (3.3 \times 10^{4})^2$$

$$K = 9.3 \times 10^{-19} \text{ Joule}$$

In nuclear physics joule is a very large energy unit

so $1 \text{ ev} = 1.6 \times 10^{-19}$ J

$$\therefore K = \frac{9.3 \times 10^{-19}}{1.6 \times 10^{-19}} = 5.8 \text{ ev}$$

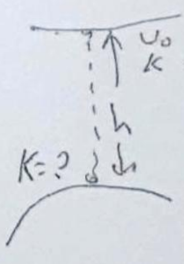
Examp 5

Assume the force of gravity to be const. for small distances above the surface of the earth. A body is dropped from rest at a height h above the earth's surface. what will the K.E be just before it strikes the ground?

particle droped from rest

work done by gravity is

$$W = \vec{F} \cdot \vec{d} = mgh$$



The gain in (K) is $\frac{1}{2} m v^2 - \frac{1}{2} m v_0^2$

$= \frac{1}{2} m v^2 - 0 \qquad W = \Delta k$

$K = mgh = \frac{1}{2} m v^2$

∴ الطاقة الحركية للجسم عند وصوله سطح الارض تساوي الشغل المبذول

$v^2 = 2gh$

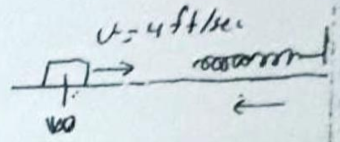
$v = \sqrt{2gh} \quad \Rightarrow \quad K.E = \frac{1}{2} m v^2 = \frac{1}{2} m (2gh) = mgh = W$

Example 6 : A block weighing 8.0 lb slides on a horizontal frictionless table with a speed of 4.0 ft/sec. It is brought to rest in compressing a spring in its path. By how much is the spring compressed if its force cons. is 0.25 lb/ft ?

2014

K.E of the block is

$K = \frac{1}{2} m v^2 = \frac{1}{2} \left(\frac{W}{g}\right) v^2$



K is equal to W that the block can do before it is brought to rest.

$W = \frac{1}{2} k x^2$ السبرنك

The work done in compressing the spring a distance x beyond its unstretched length is

$$W = \frac{1}{2} k x^2$$

So that

$$\frac{1}{2} k x^2 = \frac{1}{2} \frac{W}{g} v^2$$

$$x = \sqrt{\frac{W}{gk}}\, v = \sqrt{\frac{8.0}{32 \times 0.25}} \times 4.0$$

$x = 4.0$ ft.

Scanned by CamScanner

power: we define power as the time rate at which work is done. المعدل الزمني للشغل المنجز

time rate of doing work :- instantaneous

$$P = \frac{dw}{dt} \Longrightarrow \text{average power}$$

If the power is constant in time, then,

$$w = pt \;, \quad p = \frac{w}{t} = \frac{F \cdot d}{t} \Leftarrow \text{average power}$$

Unit of power is 1 Joule/sec ⇒ watt in mks system. In British system was lb-ft/sec this unit is quite small for practical purposes a larger unit called horsepower

1 horse power = 550 ft-lb/sec

1 horse power = 746 watts or $\frac{3}{4}$ k watt

قانون آخر للقدرة الذي تتعلق بسرعة الجسم ⇐

problems 1, 2, 3, 4, 5, 9, 10, 11, 12, 15, 21

For instantanous power: If the direction of force $\vec{F}$ is at angle ϕ to the direction of travel of the object then :*

$$\left\{ \begin{array}{l} \Delta w = F \cdot \Delta x \\ dw = F \cdot dx = F\cos\phi \, dx \end{array} \right.$$

$$* \; P = \frac{dw}{dt} = \frac{F\cos\phi \, dx}{dt} = F\cos\phi \frac{dx}{dt}$$

$$P = FV\cos\phi \Longrightarrow P = \vec{F} \cdot \vec{v}$$

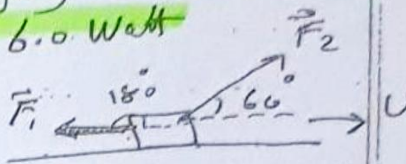
(42)

Example: Two constant forces $\vec{F}_1$ and $\vec{F}_2$ acting on a box slides rightward across a frictionless floor. Force $\vec{F}_1$ is horizontal, with magnitude 2.0 N.; $\vec{F}_2$ is angled upward by 60° to the floor and has magnitude 4.0 N. The speed v of the box at a certain instant is 3.0 m/s

لحظة معينة

a) what is the power due to each force acting on the box at that instant and what is the net power? Is the net power changing at that instant?

$P_1 = F_1 v \cos\phi_1 = 2 \times 3 \cos 180 = -6.0$ Watt



This negative result tells us that force $\vec{F}_1$ is transferring energy from the box at the rate of 6 J/s

الاشارة السالبة تعني ان القوة F_1 تنقل طاقة من الجسم بمعدل ٦ واط .

$P_2 = F_2 v \cos\phi_2 = 4 \times 3 \cos 60 = 6.0$ Watt

هذه النتيجة الموجبة تعني ان القوة F_2 تنقل طاقة الى الجسم بمعدل ٦ واط (b

This positive result tells us that force F_2 is transferring energy to the box at the rate of 6 J/s.

$P_{net} = P_1 + P_2 = -6 + 6 = 0$

هذا تعني معدل الطاقة المنقولة الى الجسم هو صفر

اي ان الطاقة الحركية للجسم لا تتغير . اي ان سرعة الجسم تبقى ثابتة 3 m/s

b) If the magnitude of F_2 is 6.0 N. What is the net power and is it changing? $P_2 = 6 \times 3 \cos 60 = 9.0$ watt

$\therefore P_{net} = -6 + 9 = 3.0$ W i.e. K.E → increases and v also increase

and P_{net} is changing.