

التابع، المجال

التاريخ / / ٢٠١

الموضوع :

The Domain

المجال

Let S and T denoted sets, a function f

from S to T is relationship (formula rule) that assigns one element of T to each element in S ,

The sets is called the ~~Domain~~ Domain.

① Constant function

المجال $f(x) = k$, $k \in \mathbb{R}$ مجموعة الأعداد الحقيقية

$f(x) = 3$, $f(x) = \frac{1}{6}$, $f(x) = -1.2$

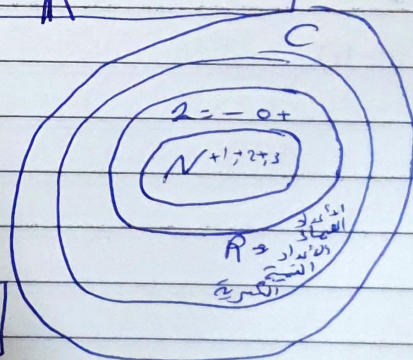
$f(x) = \sqrt{5}$, $f(x) = 0.29$

المجال $f(x) = 1.2$ مجموعة الأعداد الحقيقية

$f(x) = 1.2$ المجال $f(x) = 1.2$

$D_f = \mathbb{R}$

$f(x) = 1.10 \rightarrow D_f = \mathbb{R}$



الأعداد الحقيقية هي الأعداد التي تكون صحيحة، أو كسرية، أو غير صحيحة
 $\sqrt{2} = 1.7$
 $\frac{1}{3} = 0.3$

② Absolute Value Function (القيمة المطلقة)

$$F(x) = |x| = \begin{cases} x & , x \geq 0 \\ -x & , x < 0 \end{cases}$$

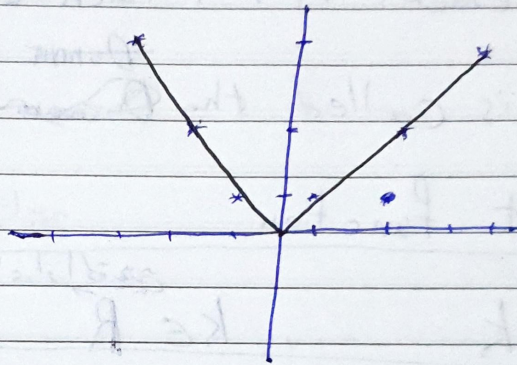
المعرف العام
للقيمة المطلقة

$$|x| = |b|$$

$$D_f = \mathbb{R} \in \text{القيمة المطلقة}$$

↗ +3
↘ -3

x	y
-3	3
-2	2
-1	1
0	0
1	1
2	2
3	3



③ Polynomial function.

دالة متعددة الحدود

$$f(x) = ax^n + bx^{n-1} + cx^{n-2} + \dots + z$$

المصطلح الثابت
a, b, c
constant

مثال

Example:-

دالة تربيعية

$$f(x) = 3x^2 + 2x - 5$$

دالة

$$f(x) = 5x^3 - 2x^2 + 2x - 1$$

$$D_f = \mathbb{R}$$

④ Fractional Function

الدالة الكسرية

$$f(x) = \frac{h(x)}{g(x)} \quad \text{where } g(x) \neq 0$$

$$D_f = \mathbb{R} / g(x) = 0$$

Ex

$$f(x) = \frac{3x}{x+5}$$

$$x+5=0 \Rightarrow x=-5$$

$$D_f = \mathbb{R} - \{-5\}$$

نقطة

$$\frac{-15}{0}$$

Calculus

٢٠١ / /

التاريخ

الموضوع :

Ex

$$f(x) = \frac{x^2 + 3x - 1}{x^2 - 6x + 5}$$

$$x^2 - 6x + 5 = 0$$

$$(x - 5)(x - 1) = 0$$

$$x_1 = 5$$

$$x_2 = 1$$

أقواس ممتدة

$$\Rightarrow D_f = \mathbb{R} - \{1, 5\}$$

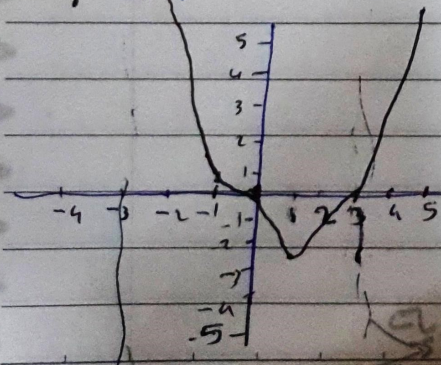
Example

$$f(x) = \frac{4x^2 + 2x - 1}{|x| - 3}$$

$$|x| - 3 = 0$$

$$\Rightarrow |x| = 3, \quad x = \pm 3$$

$$D_f = \mathbb{R} - \{-3, 3\}$$



x	y
-4	55 (-4, 55)
-2	-11 (-2, -11)
-1	-1/2 (-1, -1/2)
0	1/3 (0, 1/3)
1	-5/2 (1, -2.5)
2	-19 (2, -19)
4	71 (4, 71)

CLEAR

لذا لا نأخذ قيمتين

$$F(x) = \frac{4x^2 - 2x + 3}{x^3 + 2x^2 - x - 2}$$

$$x^3 + 2x^2 - x - 2 = 0$$

$$(x-1)(x^2+3x+2) = 0$$

$$(x-1)(x+1)(x+2) = 0$$

$$x = 1, -1, -2$$

$$D_f = \mathbb{R} - \{1, -1, -2\}$$

$$(x^3 + 2x^2) - (x-2) = 0$$

$$x^2(x+2) - (x+2) = 0$$

$$(x+2)(x^2-1) = 0$$

$$(x+2)(x+1)(x-1) = 0$$

$$x_1 = -2$$

$$x_2 = -1$$

$$x_3 = 1$$

$$D_f = \mathbb{R} - \{-2, -1, 1\}$$

② $\text{root} = 1$ $x - \text{root}$ $x^2 + 3x + 2$

$$\begin{array}{r} (x-1) \overline{) x^3 + 2x^2 - x - 2} \\ \underline{x^3 - x^2} \\ 3x^2 - x \\ \underline{3x^2 - 3x} \\ 2x - 2 \\ \underline{2x - 2} \\ 0 \end{array}$$

$$x^3 + 2x^2 - x - 2 = 0 \Rightarrow (x-1)(x^2 + 3x + 2)$$

$$\Rightarrow (x-1)(x+1)(x+2)$$

$$x_1 = 1$$

$$x_2 = -1$$

$$x_3 = -2$$

Rational function

الدوال النسبية

* even power, $\sqrt[n]{a}$, $\sqrt[n]{a^2}$, $\sqrt[n]{a^3}$, ...

$$\text{Degree} \geq 0$$

NOTE

$$① \sqrt{x} \leq a \Rightarrow x^2 \leq a^2 \Rightarrow -a \leq x \leq a$$

$$② \sqrt{x} < a \Rightarrow x^2 < a^2 \Rightarrow -a < x < a$$

$$③ \sqrt{x} \geq a \Rightarrow x^2 \geq a^2 \Rightarrow x \geq a \text{ or } x \leq -a$$

$$④ \sqrt{x} > a \Rightarrow x^2 > a^2 \Rightarrow x > a \text{ or } x < -a$$

Example

$$f(x) = \sqrt{x-5}$$

$$x-5 \geq 0 \Rightarrow x \geq 5$$

$$\Rightarrow D_f = \{x : x \in \mathbb{R}, x \geq 5\}$$

$\sqrt{1-x}$

$$f(x) = \sqrt{1-x}$$

$$1-x \geq 0$$

$$\Rightarrow 1 \geq x \Rightarrow x \leq 1$$

$$\Rightarrow D_f = \{x: x \in \mathbb{R}, x \leq 1\}$$

x	f(x)
2	$\sqrt{3}$
1	$\sqrt{2}$
0	1
-1	0
-2	$\sqrt{3}$

Ex $f(x) = \sqrt{27 - 3x^2}$

$$27 - 3x^2 \geq 0 \Rightarrow 27 \geq 3x^2$$

$$9 \geq x^2 \Rightarrow x^2 \leq 9 \quad (1) \text{ (مربعين لا)}$$

$$-3 \leq x \leq 3$$

$$D_f = \{x: x \in \mathbb{R}, -3 \leq x \leq 3\}$$

x	f(x)
-3	0
-2	3
-1	6
0	9
1	6
2	3
3	0

Ex: $f(x) = \sqrt{x^2 - 16}$

$$x^2 - 16 \geq 0$$

$$x^2 \geq 16$$

$$x \geq 4$$

$$\text{or } x \leq -4$$

$$D_f = \{x: x \in \mathbb{R}, x \geq 4 \text{ or } x \leq -4\}$$

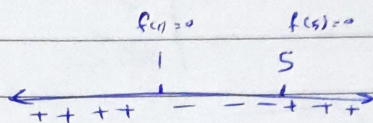
Ex) $f(x) = \sqrt{x^2 - 6x + 5}$

$$x^2 - 6x + 5 \geq 0$$

boundary point

roots = 1, 5

$$(x-1)(x-5) \geq 0$$



$$D_f = \{x: x \in \mathbb{R}, x \leq 1, x \geq 5\}$$

$$(-\infty, 1] \cup [5, +\infty)$$

$$R = [2, 4]$$

* ^{فردية} odd power $\sqrt[3]{g(x)}, \sqrt[5]{g(x)}, \sqrt[7]{g(x)}$

$$D_g = \mathbb{R}$$

Ex

$$① \sqrt[3]{x^3 - 1}$$

جميع الأعداد الحقيقية

$$\Rightarrow D_f = \mathbb{R}$$

$$② \sqrt[3]{x^2 - 2x + 4}$$

$$f_x: f(x) = \frac{1}{\sqrt{3-x}}$$

$$\sqrt{3-x} > 0$$

$$\Rightarrow 3-x > 0 \Rightarrow x < 3$$

$$D_f = \{x: x \in \mathbb{R}, x < 3\}$$

$$f_x$$

$$1) f(x) = \frac{x^2+2x-5}{\sqrt{x-4}}$$

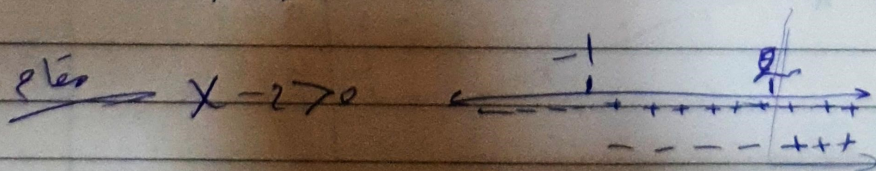
$$x-4 > 0 \Rightarrow x > 4$$

$$D_f = \{x: x \in \mathbb{R}, x > 4\}$$

$$2) f(x) = \frac{\sqrt{x+1}}{\sqrt{x-2}}$$

$$x+1 \geq 0$$

$$\Rightarrow x \geq -1$$



$$\Rightarrow x > 2$$

$$D_f = \{x: x \in \mathbb{R}, x > 2\}$$

The Range

المجموعة (المجال) الناتجة

The set of out put (y-value) that result when x varies over the domain is called the range of f .

$$D_f = \{ \}$$

$$R_f = \{ \}$$

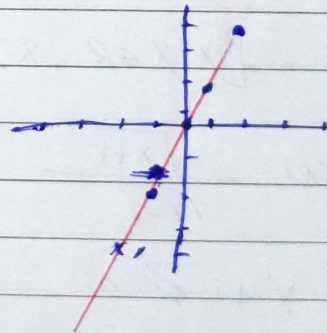
نصف المجال

Ex ① $f(x) = 2x$

$$\Rightarrow D_f = \mathbb{R}$$

$$\Rightarrow R_f = \mathbb{R}$$

x	$f(x)$
-2	-4
-1	-2
0	0
1	2
2	4

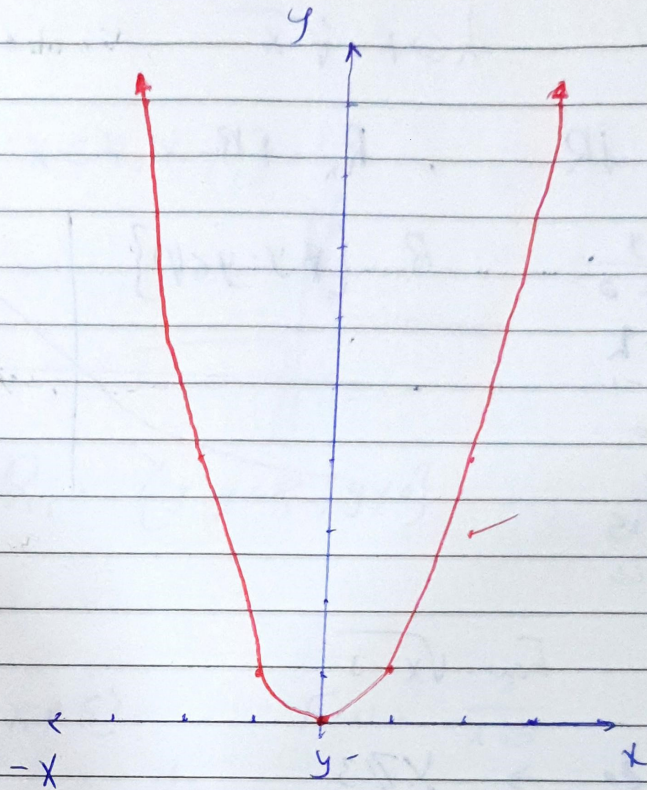


②

$$f(x) = x^2$$

$$D_f = \mathbb{R}$$

x	$f(x)$
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

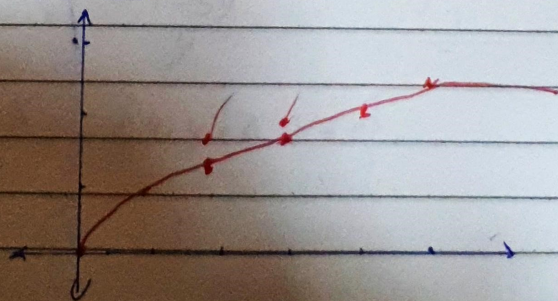


$$R_f = \{y: y \in \mathbb{R}, y \geq 0\}$$

$$(3) g(x) = \sqrt{x}$$

$$D_g = \{x: x \in \mathbb{R}, x \geq 0\}$$

x	y
0	0
1	1
2	1.41
3	1.73
4	2
5	2.23



$$R_g = \{y: y \in \mathbb{R}, y \geq 0\}$$

②

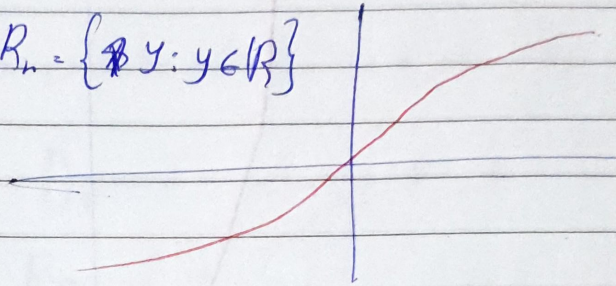
$$h(x) = \sqrt[3]{x}$$

$$D_h = \mathbb{R}$$

$$R_h = \mathbb{R}$$

x	y
-27	-3
-8	-2
-1	-1
0	0
1	1
2	1.25
3	1.44

$$R_h = \{y : y \in \mathbb{R}\}$$



⑤

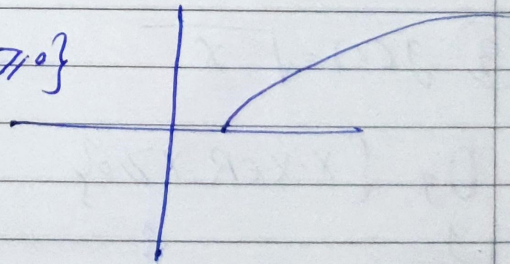
$$f(x) = \sqrt{x-3}$$

$$x-3 \geq 0 \Rightarrow x \geq 3$$

$$\Rightarrow D_f = \{x : x \in \mathbb{R}, x \geq 3\}$$

x	y
3	0
4	1
5	1.41
6	1.7
7	2
8	2.3
9	2.4

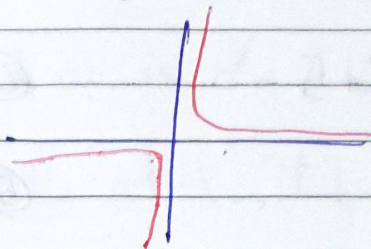
$$\Rightarrow R_f = \{y : y \in \mathbb{R}, y \geq 0\}$$



$$h(x) = \frac{1}{x}$$

$$D_h = \{x: x \in \mathbb{R}, x \neq 0\}$$

x	y
-3	-1/3
-2	-1/2
-1	-1
1	1
2	1/2
3	1/3



$$R_h = \{y: y \in \mathbb{R}, y \neq 0\}$$

$$\textcircled{7} \quad D_f = \{x: x \in \mathbb{R}, x \neq -5\}$$

$$F(x) = \frac{1}{x+5}$$

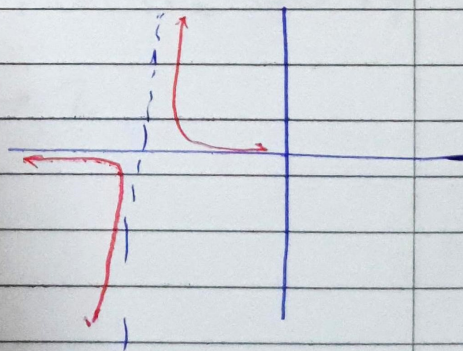
x	y
-6	-1
-4	1
-3	1/2
-2	1/3
-1	1/4
0	1/5
1	1/6
2	1/7

$$y = \frac{1}{x+5}$$

$$(x+5) = \frac{1}{y}$$

$$\Rightarrow x = \frac{1}{y} - 5$$

$$x = \frac{1-5y}{y}$$



$$\Rightarrow R_f = \{y: y \in \mathbb{R}, y \neq 0\}$$

H.W:- Find Domain, Range and Sketch the following function:-

① $F(x) = 2x^2 + 5$

④ $h(x) = |x - 2|$

② $f(x) = x^3$

⑤ $g(x) = x^{\frac{2}{3}} = \sqrt[3]{x^2}$

③ $f(x) = \frac{1}{x-3}$

⑥ $g(x) = |x| - 2$

Limits : القابل

Def: Let X_0 be a point in some interval I , and let $f(x)$ be a function which is defined at every point of I except possibly at X_0 , the limit of the function at X_0 is L , written $\lim_{x \rightarrow X_0} f(x) = L$

Ex: Find the value of the function

$f(x) = 2x + 3$ at $x = 2$ then find the limit of the ~~limit~~ function when $x \rightarrow 2$

$$f(2), 2(2) + 3 = 7$$

$$\lim_{x \rightarrow 2} 2x + 3 = 2(2) + 3 = 7$$

Ex: what will happen for the function

$$f(x) = \frac{x^2 - 9}{x - 3} \text{ when } x \rightarrow 3?$$

$$\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3} = \lim_{x \rightarrow 3} \frac{(x-3)(x+3)}{(x-3)} \quad \text{الحدس}$$

$$\lim_{x \rightarrow 3} = x + 3 = 6$$

Properties of Limits

If $\lim_{x \rightarrow c} f(x) = L_1$, $\lim_{x \rightarrow c} g(x) = L_2$ then

$$① \lim_{x \rightarrow c} [f(x) + g(x)] = \lim_{x \rightarrow c} f(x) + \lim_{x \rightarrow c} g(x) = L_1 + L_2$$

$$② \lim_{x \rightarrow c} [f(x) - g(x)] = \lim_{x \rightarrow c} f(x) - \lim_{x \rightarrow c} g(x) = L_1 - L_2$$

$$③ \lim_{x \rightarrow c} [f(x) \cdot g(x)] = \lim_{x \rightarrow c} f(x) \cdot \lim_{x \rightarrow c} g(x) = L_1 \cdot L_2$$

$$④ \lim_{x \rightarrow c} [k f(x)] = k \lim_{x \rightarrow c} f(x), k \in \mathbb{R}$$

$$⑤ \lim_{x \rightarrow c} \left[\frac{f(x)}{g(x)} \right] = \frac{\lim_{x \rightarrow c} f(x)}{\lim_{x \rightarrow c} g(x)} = \frac{L_1}{L_2}, L_2 \neq 0$$

$$⑥ \lim_{x \rightarrow c} [f(x)]^s = (L_1)^s, s \neq 0$$

Ex $f(x) = x^2 + 2x + 4, x \rightarrow 2$

$$g(x) = x + 2$$

لـ $\frac{f(x)}{g(x)}$

$$\lim_{x \rightarrow 2} \frac{x^2 + 2x + 4}{x + 2} = \frac{12}{4} = 3$$

$$\lim_{x \rightarrow 2} x^2 + 2x + 4 = 4 + 4 + 4 = 12 = L_1 \text{ (المقام)}$$

$$\lim_{x \rightarrow 2} x + 2 = 2 + 2 = 4 = L_2$$

$$\Rightarrow \frac{L_1}{L_2} = \frac{12}{4} = 3$$

Note

$$(*) \text{ If } f(x) = k \Rightarrow \lim_{x \rightarrow c} k = k$$

$$(*) \text{ If } f(x) = x \Rightarrow \lim_{x \rightarrow c} f(x) = c \quad \begin{matrix} f(x) = x \\ \lim_{x \rightarrow 10} f(x) = 10 \end{matrix}$$

$$(*) \text{ If } f(x) = ax^2 + bx + k$$

$$\Rightarrow \lim_{x \rightarrow c} f(x) = ac^2 + bc + k$$

Definition

* Right hand limit

$$\lim_{x \rightarrow c^+} f(x) = L, \text{ means that } f(x) \rightarrow L$$

as $x \rightarrow c$ From right

* Left hand limit

$$\lim_{x \rightarrow c^-} f(x) = L, \text{ means that } f(x) \rightarrow L, \text{ as } x \rightarrow c$$

From left

Limit exist

$\lim_{x \rightarrow c} f(x)$ exist and equal to L if

$$\lim_{x \rightarrow c^-} f(x) = \lim_{x \rightarrow c^+} f(x) = L$$

Ex $\uparrow f$ $f(x) = \begin{cases} x^2 - 2, & x \geq 2 \\ x, & x < 2 \end{cases}$

القيمة من اليمين

$$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} x^2 - 2 = (2) = L_1$$

القيمة من اليسار

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} x = (2) = L_2$$

$\Rightarrow$ Limit is exist
 $L_1 = L_2$

Ex Find $\lim_{x \rightarrow 0} \frac{|x|}{x}$

$$\therefore |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$f(x) = \begin{cases} \frac{x}{x} = 1, & x \geq 0 \\ \frac{-x}{x} = -1, & x < 0 \end{cases}$$

$$\Rightarrow \lim_{x \rightarrow 0^+} \frac{x}{x} = \lim_{x \rightarrow 0^+} 1 = 1 = L_1$$

$$\lim_{x \rightarrow 0^-} \frac{-x}{x} = \lim_{x \rightarrow 0^-} -1 = -1 = L_2$$

$\Rightarrow L_1 \neq L_2 \rightarrow$ Limit is not exist

Ex (D) Find $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2}$

$$\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} = \lim_{x \rightarrow 2} \frac{(x+2)(x-2)}{(x-2)} = 4$$

$$\textcircled{2} \lim_{x \rightarrow 1} \frac{x-1}{\sqrt{x}-1} = \lim_{x \rightarrow 1} \frac{(\cancel{\sqrt{x}-1})(\sqrt{x}+1)}{\cancel{\sqrt{x}-1}} = ?$$

خاله
فأصبح

Ex

$$\lim_{x \rightarrow 0} \frac{\sqrt{x+9} - 3}{x}$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{x+9} - 3}{x} \cdot \frac{\sqrt{x+9} + 3}{\sqrt{x+9} + 3}$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{x + \cancel{9} - 9}{x(\sqrt{x+9} + 3)} = \lim_{x \rightarrow 0} \frac{x}{x(\sqrt{x+9} + 3)} = \frac{1}{6}$$

L'Hôpital's Rule قاعدة لوبيتال

Suppos that f and g are differentiable

function in an open interval containing $x=a$,

excepte possibly at $x=a$, and that

$$\lim_{x \rightarrow a} f(x) = 0 \quad \text{and} \quad \lim_{x \rightarrow a} g(x) = 0$$

If $\lim_{x \rightarrow a} \left[\frac{f'(x)}{g'(x)} \right]$ has fin limit, then

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

Moreover, this statement is also true in the case and a limit.

$$\text{as } x \rightarrow a^-$$

$$x \rightarrow a^+$$

$$x \rightarrow -\infty$$

$$x \rightarrow +\infty$$

Simillary $\lim_{x \rightarrow a} f(x) = +\infty$ or $-\infty$

gR $\lim_{x \rightarrow a} g(x) = +\infty$ or $-\infty$

Examples

$$\textcircled{1} \lim_{x \rightarrow 2} \frac{x^2 - 4}{x^2 + 2x - 8} = \lim_{x \rightarrow 2} \frac{(x-2)(x+2)}{(x-2)(x+4)}$$

$$= \lim_{x \rightarrow 2} \frac{x+2}{x+4} = \frac{4}{6} = \frac{2}{3}$$

$$\textcircled{2} \lim_{x \rightarrow 0} \frac{\cos x - 1}{\sin x}$$

$$\lim_{x \rightarrow 0} \frac{-\sin x}{\cos x} = \frac{0}{1} = 0$$

$$\textcircled{1} \lim_{x \rightarrow 0} \frac{e^x - 1}{\sin x} = \lim_{x \rightarrow 0} \frac{e^x}{\cos x} = \frac{e^0}{1} = \frac{1}{1} = 1$$

the problem is $\infty - \infty$ is not counted

$$\textcircled{2} \lim_{x \rightarrow \infty} [\sqrt{x^2 + x} - x]$$

المشكلة هي $\infty - \infty$ لا تحسب
بالنسبة

$$\Rightarrow \lim_{x \rightarrow \infty} \frac{[\sqrt{x^2 + x} - x][\sqrt{x^2 + x} + x]}{[\sqrt{x^2 + x} + x]}$$

$$\Rightarrow \lim_{x \rightarrow \infty} \frac{x^2 + x - x^2}{[\sqrt{x^2 + x} + x]} = \lim_{x \rightarrow \infty} \frac{x}{\sqrt{x^2 + x} + x}$$

نقسم البسط والمقام
على x

$$\Rightarrow \lim_{x \rightarrow \infty} \frac{x/x}{\frac{\sqrt{x^2 + x} + x}{x}} = \lim_{x \rightarrow \infty} \frac{1}{\sqrt{\frac{x^2 + x}{x^2}} + \frac{x}{x}}$$

$$= \lim_{x \rightarrow \infty} \frac{1}{\sqrt{1 + \frac{1}{x}} + 1} = \frac{\lim_{x \rightarrow \infty} 1}{\lim_{x \rightarrow \infty} \sqrt{1 + \frac{1}{x}} + \lim_{x \rightarrow \infty} 1}$$

$$= \frac{1}{\sqrt{1} + 1} = \frac{1}{1 + 1} = \frac{1}{2}$$

H.W

$$\textcircled{1} \lim_{x \rightarrow 0} \frac{e^x - 1}{\sin x}$$

$$\textcircled{2} \lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{\sin x}$$

$$\textcircled{3} \lim_{x \rightarrow 1} \frac{\sqrt{x} - x}{\ln x}$$

$$(٤) \lim_{x \rightarrow 0} \frac{\sin x - x}{x^3}$$

Notes:

① إذا كانت الدالة مركبة، مجالها هو القاعدة $x=a$ ، الثانية $x \neq a$

$$f(x) = \begin{cases} 2-x & x \neq -1 \\ 3 & x = -1 \end{cases} \quad \text{أما مجال الثانية عند } x \neq -1$$

$$\lim_{x \rightarrow -1} 2-x = 2 - (-1) = 3$$

② إذا كان مجالها هو القاعدة $x > a$ ، والآخر $x < a$ عند هاتين

في قيمة الثانية من اليسار واليمين

$$f(x) = \begin{cases} x^2 + 1 & x > 1 \\ 2x & x < 1 \end{cases} \Rightarrow \lim_{x \rightarrow 1} x^2 + 1 = 2 = L_1$$

$$\lim_{x \rightarrow 1} 2x = 2 = L_2$$

$$\Leftarrow \text{الدالة غائبة عند } x=1 \text{ (} L_1 \neq L_2 \text{)}$$

Def A function f is continuous at a number

c if $\lim_{x \rightarrow c} f(x) = f(c)$, Thus the

continuity of f at c requires three things.

(1) $\lim_{x \rightarrow c} f(x) = L$ exists

(2) $f(c)$ exists

(3) $\lim_{x \rightarrow c} f(x) = f(c)$, i.e. $L = f(c)$

ex: Determine whether the following

function are continuous at x

(1) $f(x) = \begin{cases} \frac{x^2 - 4}{x - 2}, & x \neq 2 \\ 3, & x = 2 \end{cases}$

(1) $\lim$

$\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} = \lim_{x \rightarrow 2} \frac{(x+2)(x-2)}{x-2} = \lim_{x \rightarrow 2} (x+2) = 4$

$\lim_{x \rightarrow 2} f(x) = 4$

(2) $f(2) = 3$

(3) $\lim_{x \rightarrow 2} f(x) \neq f(2) \Rightarrow f$ is not continuous

تا عدد لوبيتال ميگت استه اما انكره مره.

Ex 20981 : الثانية / / ٢٠١

Example: check the continuity of the following

$$(1) f(x) = \begin{cases} 2x-3 & , x \leq 2 \\ x^2 & , x > 2 \end{cases}$$

$$(1) \lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2} x^2 = 4$$

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2} 2x-3 = 1$$

$\Rightarrow$ the limit is not exists

$$(2) f(x) = |x-2| \quad \text{at } x=2$$

$$f(x) = \begin{cases} x-2 & x > 2 \\ -(x-2) & x \leq 2 \end{cases}$$

$$(1) \lim_{x \rightarrow 2} x-2 = 0$$

$$\lim_{x \rightarrow 2} 2-x = 0$$

$\Rightarrow$ limit is exists

$$(2) f(2) = 2-2 = 0$$

$$\therefore \lim_{x \rightarrow 2} f(x) = f(2) \Rightarrow f \text{ is con.}$$

Some Properties For Continuity

① The constant function is continuous f .

$$f(x) = k, k \in \mathbb{R}$$

البرهان ① $\lim_{x \rightarrow x_0} f(x) = \lim_{x \rightarrow x_0} k = k$

② $f(x) = k$

$$\therefore \lim_{x \rightarrow x_0} f(x) = f(x_0) \Rightarrow k = k$$

$\Rightarrow f$ is con.

~~***~~ The Identity function is continuous

function

$$f(x) = x \quad (\text{Identity function})$$

$y = x$

Let f and g be functions which are continuous at c , then $f+g$ and $f-g$ are continuous at c , if $g(c) \neq 0$ then f/g is continuous at c .

المشتقة

The Derivative

Def Suppose that x_0 is a number in the domain of a function f . If

$$\lim_{x \rightarrow x_0} \frac{f(x) - f(x_0)}{x - x_0} \text{ exists, then}$$

the value of this limit is called the derivative of f at $x = x_0$ and denoted by $f'(x_0) =$, i.e.

$$f'(x_0) = \lim_{x \rightarrow x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

Note $\Delta x = x_1 - x_0$

$$x_1 = \Delta x + x_0$$

$$f'(x_0) = \lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

This called the mathematical definition

for derivate

Examples:- Find the derivative for the following

function by using definition of derivative

استخدام التعريف

① $f(x) = 3x - 4$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

قانون تعريف المشتقة

$$= \lim_{\Delta x \rightarrow 0} \frac{[3(x + \Delta x) - 4] - [3x - 4]}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{3x + 3\Delta x - 4 - 3x + 4}{\Delta x} = 3$$

② $f(x) = \sqrt{x}$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{\sqrt{x + \Delta x} - \sqrt{x}}{\Delta x} \times \frac{\sqrt{x + \Delta x} + \sqrt{x}}{\sqrt{x + \Delta x} + \sqrt{x}}$$

نضرب بالمرافقة

$$(a-b)(a+b) = a^2 - b^2$$

عدد x مرفوع = مرفوع - مرفوع

الآلية

الموضوع:

$$= \lim_{\Delta x \rightarrow 0} \frac{x + \Delta x - x}{\Delta x (\sqrt{x + \Delta x} + \sqrt{x})}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{1}{\sqrt{x + \Delta x} + \sqrt{x}}$$

$$= \frac{1}{\sqrt{x+0} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$$(3) f(x) = x^2$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x)^2 - x^2}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{x^2 + 2x\Delta x + \Delta x^2 - x^2}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{\Delta x(2x + \Delta x)}{\Delta x} = 2x$$

$$(4) f(x) = \frac{1}{x}$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{\frac{1}{x + \Delta x} - \frac{1}{x}}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{\frac{x - (x + \Delta x)}{x(x + \Delta x)}}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{\frac{-\Delta x}{x^2 + x\Delta x}}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{-\Delta x}{\Delta x (x^2 + x\Delta x)}$$

$$= \frac{-1}{x^2 + x(0)} = \frac{-1}{x^2}$$

Ex The function $f(x) = |x|$ has a corner

at $x=0$, Prove that f is not differentiable

at $x=0$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{|x+\Delta x| - |x|}{\Delta x}$$

$$f'(0) = \lim_{\Delta x \rightarrow 0} \frac{|0+\Delta x| - |0|}{\Delta x}$$

$$f'(0) = \lim_{\Delta x \rightarrow 0} \frac{|\Delta x|}{\Delta x} \begin{cases} + \frac{\Delta x}{\Delta x} & \Delta x > 0 \\ - \frac{\Delta x}{\Delta x} & \Delta x < 0 \end{cases}$$

$$\lim_{\Delta x \rightarrow 0} f(x) = +1$$

$$\lim_{\Delta x \rightarrow 0} f(x) = -1$$

Formulas for Differentiable Algebraic

functions

قواعد التفاضل الجبري

$$y = f(x) \cdot \frac{dy}{dx} = f'(x) = D_x f = \frac{d}{dx} (f(x))$$

$$① D_x (k) = 0, k \in \mathbb{R}$$

$$② D_x (x) = 1$$

$$(3) D_x (x^n) = n x^{n-1}$$

$$(4) D_x [k f(x)] = k D_x [f(x)]$$

(5) If u and v are diff. f., then

$$D_x (u \pm v) = D_x u \pm D_x v$$

$$(6) D_x (u \cdot v) = u \cdot D_x v + v \cdot D_x u$$

$$(7) D_x \left(\frac{u}{v} \right) = \frac{v \cdot D_x u - u \cdot D_x v}{v^2}$$

$x \neq 0 \Rightarrow v \neq 0$

$$(8) D_x (u)^n = n (u)^{n-1} \cdot D_x u$$

$$(9) D_x (f \circ g) = D_x f(g(x))$$

Ex If $f(x) = x^2 + 5$

and $g(x) = 3x - 2$

Find $D_x f \circ g(1)$

$$f \circ g(x) = f(g(x)) = f(3x - 2)$$

$$= (3x - 2)^2 + 5$$

$$= 9x^2 - 12x + 4 + 5$$

$$= 9x^2 - 12x + 9$$

$$D_x f(x) = 18x - 12$$

$$D_x f(x) = 6$$

M.W $f(x) = \begin{cases} x^2 + 3 & x \leq 1 \\ 2x + 2 & x > 1 \end{cases}$

① Prove that the function is continuous at $x=1$

② Is the function $f(x)$ differentiable?

$$f(x) = |x - 3|$$

① Prove that the function is continuous at $x=3$

② Is the function $f(x)$ differentiable?

$$f(x) = \begin{cases} x-3 & x > 3 \\ -(x-3) & x < 3 \end{cases}$$

① $\lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} x - 3 = 0$

$\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^-} 3 - x = 0 \Rightarrow \text{limit exists}$

$$** f(3) = x - 3 = 3 - 3 = 0$$

$$** \lim_{x \rightarrow 3} f(x) = f(3) \Rightarrow f \text{ is cont.}$$

$$(2) f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

المجال $\lim_{\Delta x \rightarrow 0} \frac{(x+\Delta x) - 3 - (x-3)}{\Delta x}$

$$\lim_{\Delta x \rightarrow 0} \frac{x+\Delta x - 3 - x + 3}{\Delta x} = 1 = L_1$$

المجال $\lim_{\Delta x \rightarrow 0} \frac{3 - (x+\Delta x) - (3-x)}{\Delta x}$

$$= \lim_{\Delta x \rightarrow 0} \frac{3 - x - \Delta x - 3 + x}{\Delta x}$$

$$= -1 = L_2$$

$\Rightarrow f$ is not diff

* The Natural ~~Logarithm~~ Logarithm function
 (اللوغاريتم الطبيعي),
 is the function define as

$$\ln(x) = \int \frac{1}{t} dt, x > 0$$

It's domain is the Set of all positive number

The derivative of N.L.F

If $f(x) = \ln(x)$, then

$$\frac{d}{dx} \ln(x) = \frac{1}{x}$$

$$\text{in general form } \frac{d}{dx} \ln(u) = \frac{1}{u} \cdot \frac{du}{dx}$$

Theorem if a and b are positive number

and n is any rational, then

$$1. \ln(1) = 0$$

$$2. \ln e = 1, e = 2.718 \dots$$

$$3. \ln a \cdot b = \ln a + \ln b$$

$$4. \ln a \cdot b \cdot c \dots z = \ln a + \ln b + \dots + \ln z$$

$$5- \ln \frac{a}{b} = \ln a - \ln b \rightarrow = (\ln(a \cdot b))$$

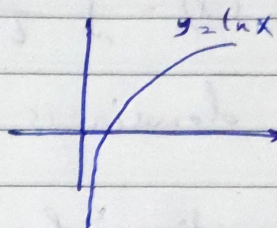
$$= \ln a - \ln b$$

$$= \ln a - \ln b$$

$$6- \ln \frac{1}{a} = -\ln a$$

$$7- \ln x^n = n \ln x$$

8- graph of $y = \ln x$ is



Examples Find $\frac{dy}{dx}$

$$① y = \ln(2x+7) \Rightarrow y' = \frac{1}{2x+7} \cdot (2)$$

$$② y = \ln(x^2+8) \Rightarrow y' = \frac{1}{x^2+8} \cdot (2x)$$

$$③ y = \ln(\ln x) \Rightarrow y' = \frac{1}{\ln x} \cdot \frac{1}{x} = \frac{1}{x \ln x}$$

$$④ y = \ln \sqrt{x} \Rightarrow y' = \frac{1}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{2x}$$

(H.W) Find y'

$$y = \ln \frac{x^3(2x+1)^{1/5}}{(x^2+1)^{1/2}}$$

* The exponential function الدالة الأسية

$$x = \ln y \Rightarrow y = e^x$$

Properties of exp. f.

1- $e^0 = 1$

2- $e' = e$, $e = 2.718$

3- $e^a \cdot e^b = e^{a+b}$, $\forall a, b \in \mathbb{R}$

4- $(e^a)^b = e^{ab}$

5- $\frac{e^a}{e^b} = e^{a-b}$

6- $e^{-a} = \frac{1}{e^a}$

7- $e^{\ln a} = a$

8- $\ln e^a = a$

The derivative of exp. f

$$\frac{d}{dx} e^x = e^x \text{ in general}$$

$$\frac{d}{dx} e^u = e^u \cdot \frac{du}{dx}$$

Example

$$① \quad y = e^{3x} \Rightarrow y' = e^{3x} \cdot (3)$$

$$② \quad y = e^{-x} \Rightarrow y' = -e^{-x}$$

$$③ \quad y = x \cdot e^x \Rightarrow y' = x \cdot e^x + e^x \cdot (1)$$

~~$$④ \quad y = e^{x^3} \Rightarrow y' = e^{x^3} \cdot (3x^2)$$~~

$$④ \quad y = e^{x^3} + \ln(\ln x) + x^3 + e$$

$$\Rightarrow y' = e^{x^3} (3x^2) + \frac{1}{\ln x} \cdot \frac{1}{x} + 3x^2 + 0$$

$$⑤ \quad y = e^{\ln x^2} = x^2 \Rightarrow y' = 2x$$

* The general exp. f. العام في الدالة

$$f(x) = a^x, \quad a \in \mathbb{R}$$

Properties of general exp. f.

$$① \quad a^0 = 1$$

$$② \quad a^1 = a$$

$$③ \quad a^x \cdot a^y = a^{x+y}$$

$$(4) a^{-x} = \frac{1}{a^x}$$

$$(5) a^x / a^y = a^{x-y}$$

$$(6) (a^x)^y = a^{x \cdot y}$$

The derivative

$$\frac{d}{dx} a^u = a^u \cdot \ln a \cdot \frac{du}{dx}$$

Example: Find $\frac{dy}{dx}$

$$(1) y = 7^{x^2} \Rightarrow y' = 7^{x^2} \cdot \ln 7 \cdot (2x)$$

$$(2) y = e^{5x^2} \cdot 7^{3x^2}$$

$$\Rightarrow y' = e^{5x^2} \cdot [7^{3x^2} \cdot \ln 7 \cdot (6x)] + 7^{3x^2} \cdot [e^{5x^2} \cdot (10x)]$$

$$(3) y = e^{\ln(x^2+5x)} \cdot 8^{x^2}$$

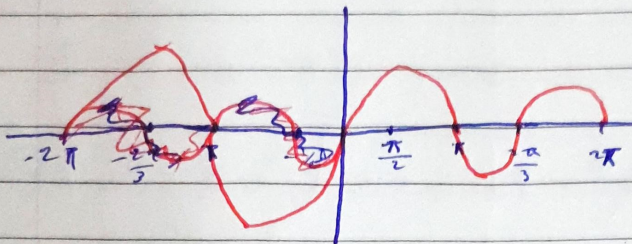
$$\Rightarrow y' = (x^2+5x) \cdot [8^{x^2} \cdot \ln 8 \cdot (2x)] + 8^{x^2} [2x+5]$$

ل.ل.

1 / 15 Trigonometric function الدالة: Sinusoidal

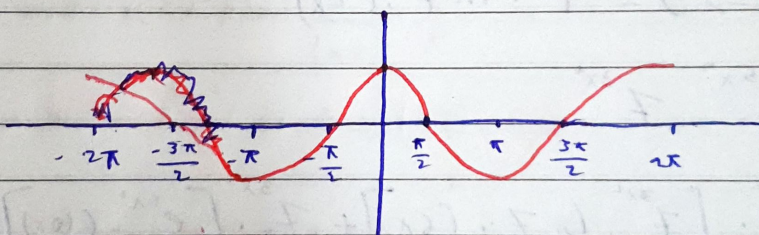
① $y = \sin x$

$D_y = \mathbb{R}, R_y = [-1, 1]$



② $y = \cos x$

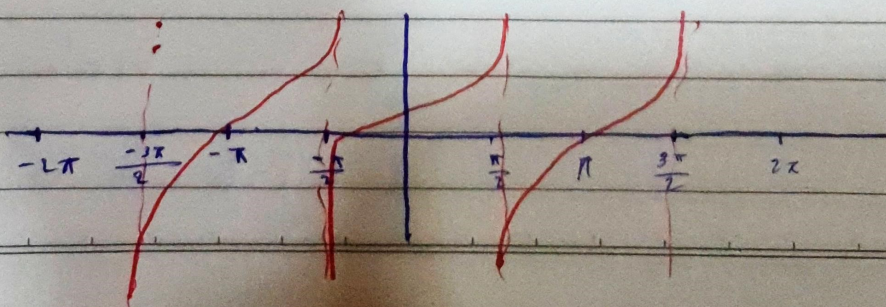
$D_y = \mathbb{R}, R_y = [-1, 1]$



③ $y = \tan x$

$D_y = \mathbb{R} - \left\{ \frac{(2n-1)\pi}{2}, n \in \mathbb{N} \right\}$

$R_y = (-\infty, \infty)$

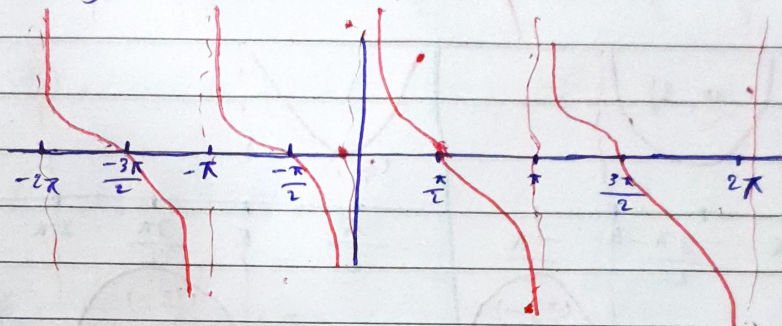


CLEAR

④ $y = \cot x$

$$D_y = \mathbb{R} / \{(n-1)\pi, n \in \mathbb{N}\}$$

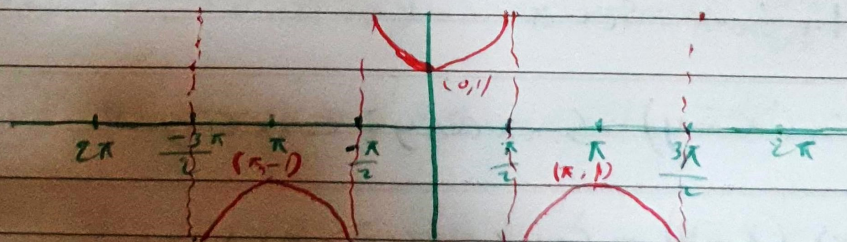
$$R_y = (-\infty, +\infty)$$



⑤ $y = \sec x$

$$D_y = \mathbb{R} / \left\{ \frac{(2n-1)\pi}{2}, n \in \mathbb{N} \right\}$$

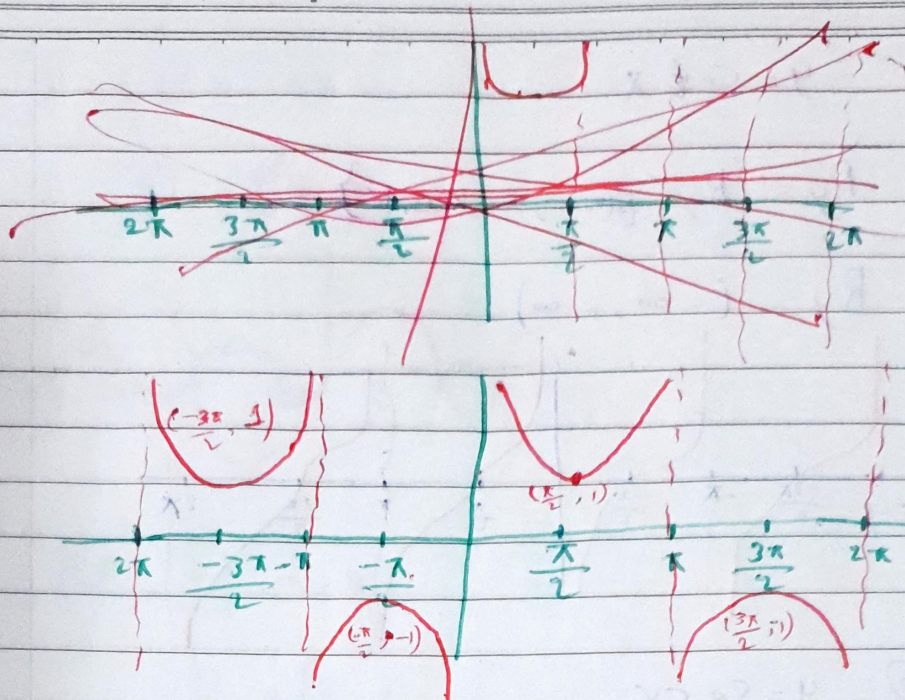
$$R_y = (-\infty, -1] \cup [1, +\infty)$$



⑥ $y = \csc x$

$$D_y = \mathbb{R} / \{(n-1)\pi, n \in \mathbb{N}\}$$

$$R_y = (-\infty, -1] \cup [1, +\infty)$$



Some Rules

- ① $\sin^2 x + \cos^2 x = 1$
- ② $1 + \tan^2 x = \sec^2 x$
- ③ $\sin(x+y) = \sin x \cos y + \cos x \sin y$
- ④ $\cos(x+y) = \cos x \cos y - \sin x \sin y$
- ⑤ $\sin 2x = 2 \sin x \cos x$
- ⑥ $\cos 2x = \cos^2 x - \sin^2 x$
 $= 1 - 2 \sin^2 x$

$$-2\cos^2 x - 1$$

$$\textcircled{7} \sin \frac{\textcircled{2} X}{\textcircled{2}} = \frac{1 - \cos X}{2} \quad \sin \frac{X}{2} = \sqrt{\frac{1 - \cos X}{2}}$$

$$\textcircled{8} \cos \frac{\textcircled{1} X}{\textcircled{2}} = \frac{1 + \cos X}{2} \quad \cos \frac{X}{2} = \sqrt{\frac{1 + \cos X}{2}}$$

Derivative of Tri. f

$$\textcircled{1} y = \sin u \Rightarrow y' = \cos u \cdot \frac{du}{dx}$$

$$\textcircled{2} y = \cos u \Rightarrow y' = -\sin u \cdot \frac{du}{dx}$$

$$\textcircled{3} y = \tan u \Rightarrow y' = \sec^2 u \cdot \frac{du}{dx}$$

$$\textcircled{4} y = \cot u \Rightarrow y' = -\csc^2 u \cdot \frac{du}{dx}$$

$$\textcircled{5} y = \sec u \Rightarrow y' = \sec u \tan u \cdot \frac{du}{dx}$$

$$\textcircled{6} y = \csc u \Rightarrow y' = -\csc u \cot u \cdot \frac{du}{dx}$$

Examples

find $\frac{dy}{dx}$

$$\textcircled{1} y = \sin 4x \Rightarrow y' = 4 \cos 4x$$

$$\textcircled{2} y = \tan^2 x \Rightarrow y' = 2 \tan x \sec^2 x (1x)$$

$$\textcircled{3} y = \tan^2 x \Rightarrow y' = 2 \tan x \sec^2 x (1)$$

$$(4) y = (3 + \cos 2x)^2$$

$$y' = 2(3 + \cos 2x) - 2 \sin 2x$$

$$(5) y = \ln(4 + \cos 3x)$$

$$y' = \frac{1}{4 + \cos 3x} \cdot (-3 \sin 3x)$$

$$(6) y = e^{\cot x} \Rightarrow y' = e^{\cot x} (-\csc^2 x)$$

H.W $y = x (e^{\sin x} + \ln \sec x) \Leftarrow *$

المختلصة
المرء والنز اسرع

Exhibi

~~(10/10/10)~~



④

⑤

60

Some Rules

$$\textcircled{1} \cosh x + \sinh x = e^x$$

$$\frac{e^x + e^{-x}}{2} + \frac{e^x - e^{-x}}{2} = \frac{e^x + e^{-x} - e^{-x} + e^x}{2} = \frac{2e^x}{2} = e^x$$

② $\cosh x - \sinh x = e^{-x}$

$$\frac{e^{-x} + e^x}{2} - \frac{e^x - e^{-x}}{2} = \frac{e^x + e^{-x} - e^x + e^{-x}}{2}$$

$$= \frac{2e^{-x}}{2} = e^{-x}$$

$$\sinh x = \frac{e^x - e^{-x}}{2i}$$

2.1 / / الثانية

: Example

Example Find the Value of $\cosh(0)$, $\sinh(0)$

$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$

$$\cosh(0) = \frac{e^0 + e^{-0}}{2} = \frac{1+1}{2} = 1$$

$$\sinh 0 = \frac{e^0 - e^{-0}}{2} = \frac{1-1}{2} = 0$$

$$\cosh(3) = \frac{e^3 + e^{-3}}{2}$$

$$\sinh^2 + \cosh^2 = \text{O}$$

Hyperbolic Trig Function Derivation

$$\textcircled{1} \frac{d}{dx} \sinh x = \cosh x$$

$$\textcircled{2} \frac{d}{dx} \cosh x = \sinh x$$

$$\textcircled{3} \frac{d}{dx} \tanh x = \text{sech}^2 x$$

$$\textcircled{4} \frac{d}{dx} \coth x = -\text{csch}^2 x$$

$$\textcircled{5} \frac{d}{dx} \text{sech } x = -\text{sech } x \tanh x$$

$$\textcircled{6} \frac{d}{dx} \text{csch } x = -\text{csch } x \coth x$$

CLEAR

Prove $\frac{d}{dx} \sinh x = \cosh x$

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\frac{d}{dx} \frac{e^x - e^{-x}}{2} = \frac{d}{dx} \left(\frac{1}{2} (e^x - e^{-x}) \right)$$

$$= \frac{1}{2} (e^x - (-1)e^x)$$

$$= \frac{1}{2} (e^x + e^x) = \cosh x$$

Q2 / prove $\frac{d}{dx} \cosh x = \sinh x$ نفس الشيء

Examples

Find $\frac{dy}{dx}$ to the function

① $y = \tanh x^2$

$$y' = 2x \operatorname{sech}^2 x^2$$

② $y = e^{\sinh 3x}$

$$y' = e^{\sinh 3x} \cdot \cosh 3x \cdot 3$$

③ $y = \ln (1 + \cosh 4x)$

$$y' = \frac{1}{(1 + \cosh 4x)} \sinh 4x \cdot 4$$