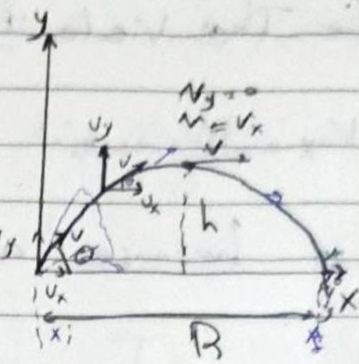
Motion in two dimension



4.3 Projectile motion

It is a curved motion. Air effect can be neglected.

The motion in two dimentional with cons. acc^n

Projectile motion can be with acc^n $a_x = 0$ in the X-direction and a cons. acc^n $a_y = -g$ in the y direction.

The projectile is projected with initial velocity v_0, when the vector v_0 makes an anghe θ with the Horizontal (i.e X-axis)

The X-component $v_{0x} = v_0 \cos\theta_0$

The y - " " $v_{0y} = v_0 \sin\theta_0$

The horizontal comp. of velocity will be cons.

(i.e The Velocity value V_x at any time is

$V_x = V_{0x} = V_0 \cos\theta_0$.

then $a_x = 0$ because acc^n ~~was~~ the change in Velocity.

The vertical Comp. of Velocity will change with time (i.e That's mean The V_y at any time can be found

$V_f = V_i + gt$

$V_{fy} = V_{0y} + gt$ $\quad\quad a_y = -g$

$\boxed{V_{fy} = V_{0y} - gt} \Rightarrow \boxed{V_{fy} = V_0 \sin\theta_0 - gt}$ — ①

$\boxed{V_x = V_{0x} = V_0 \cos\theta_0} \xRightarrow{\text{because}} V_x = V_0 \cos\theta_0 + \underline{\cancel{a_x}^{0}\, t}$

magnitude $V = \sqrt{V_x^2 + V_y^2}$

$\theta = \tan^{-1} \dfrac{V_y}{V_x}$

ⓐ Determination of time of flight

$V_y = V_{0y} + gt$

$V_y = 0$, $V_{oy} = V_o \sin\theta_o$; $g(-ve)$

$0 = V_o \sin\theta_o - gt \Rightarrow t = \frac{V_o \sin\theta_o}{g}$

t (time of maximum height)

$\boxed{T = 2t = \frac{2V_o \sin\theta_o}{g}}$ $T = \frac{2V_{oy}}{g}$

① Determination the max. height

$y = V_i t + \frac{1}{2} g t^2$

$y = y_{max} = h$; $g(-ve)$, $V_i = V_{oy}$

$h = V_{oy} t - \frac{1}{2} g t^2$

$h = V_o \sin\theta_o \left(\frac{V_o \sin\theta_o}{g}\right) - \frac{1}{2} g \left(\frac{V_o \sin\theta_o}{g}\right)^2$

$\boxed{h = \frac{1}{2} \frac{V_o^2 \sin^2\theta_o}{g}}$ $\boxed{h = \frac{1}{2}\frac{V_{oy}^2}{g}}$

$\boxed{V_f^2 = V_i^2 + 2gy}$

$\Downarrow$

$\boxed{h = \frac{1}{2} \frac{V_o^2 \sin^2\theta_o}{g}}$

H.W

(c) Detrmination the horizontal rang of projectile

$X_f = X_i + V_{xi}\, t$

$X_f - X_i = R$

$\therefore R = V_{xi} t$

$V_{xi} = V_0 \cos\theta_0$

$t = \frac{2V_0 \sin\theta_0}{g}$

$R = V_0 \cos\theta_0 \left(\frac{2V_0 \sin\theta_0}{g}\right) = \frac{V_0^2 (2\sin\theta_0 \cos\theta_0)}{g}$

$\boxed{2\sin\theta_0 \cos\theta_0 = \sin 2\theta_0}$

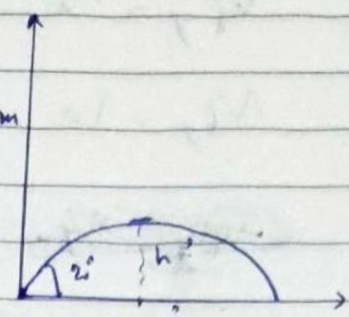
$\boxed{R = \frac{V_0^2 \sin 2\theta_0}{g}}$

$R_{max} = \frac{V_0^2}{g}$

$\sin 2\theta = 1 \; ; \; 2\theta_0 = 90^\circ \Rightarrow \theta_0 = 45^\circ$

~~P 84~~ 87
ex 4.9

$\theta_0 = 20^\circ$ $\quad v_0 = 11\ m/s$



(A) $R = \dfrac{v_0^2 \sin 2\theta_0}{g} = \dfrac{(11)^2 \sin 2(20)}{9.8} = 7.94\ m$

(B) $h = \dfrac{v_0^2 \sin^2\theta_0}{2g} = \dfrac{(11)^2(\sin(20))^2}{2(9.8)}$ $\quad V_0 = 11\ m/s \quad \theta_0 = 20^\circ$

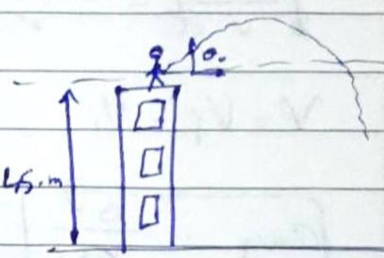
$= 0.722\ m$

Note :- How long ← طول الزمن

How far ← طول المسافة

→ ex 4.4

$\theta_0 = 30^\circ$ $\quad v_0 = 20\ m/s$



$V_{0x} = (20)(\cos 30) = 17.3\ m/s$

$V_{0y} = (20)(\sin 30) = 10.0\ m/s$

$y = V_{0y} t + \frac{1}{2} g t^2$

why is −ve ?

$-45 = 10t - \frac{1}{2}(10)t^2$ $\quad t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$t^2 - 2t - 9 = 0$

$t = \dfrac{-2 \pm \sqrt{40}}{2}$

$t = 4.25$ *

$V_{fy} = V_{iy} + gt$

$V_{fy} = 10 + (-9.8)(4.22)$

~~$V_{fy} = V_{fx}$~~ $V_{fx} = V_{ox} = 17.3\, m/s$

$V_f = \sqrt{V_{fx}^2 + V_{fy}^2} = \sqrt{(17.3)^2 + (10)^2} = 35.8\, m/s$

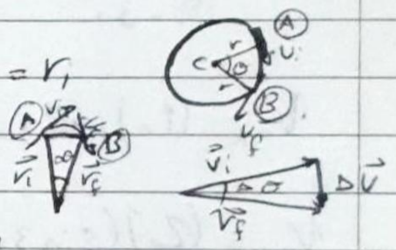
Page 91

4.4 uniform Circular motion

from similar two trigls

$\frac{|\Delta \vec{v}|}{v} = \frac{|\Delta \vec{r}|}{r} \Rightarrow |\Delta \vec{v}| = \frac{v|\Delta \vec{r}|}{r}$

$v = v_f = v_i$ $\quad$ $r = r_f = r_i$



$\boxed{a_{avg} = \frac{\Delta \vec{v}}{\Delta t}}$

$|a_{avg}| = \frac{|\Delta \vec{v}|}{|\Delta t|} = \frac{v}{r}\frac{|\Delta \vec{r}|}{\Delta t} = \frac{v^2}{r}$

$a_{avg} = \frac{v^2}{r}$

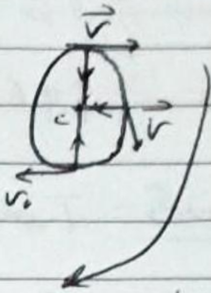
when $\Delta t \to 0$ $\quad$ $a_{avg} = a_c \Rightarrow \boxed{a_c = \frac{v^2}{r}}$

$\boxed{a_c : \text{Centripetal acc}^{n.}}$

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$

$$a = \frac{dv}{dt} = \frac{v^2}{r}$$

$\boxed{a = \frac{v^2}{r}}$ in uniform Circular motion



$$T = \frac{2\pi r}{v}$$

الانطلاق الزاوي $\omega = \frac{2\pi}{T}$ rad/sec

motion clockwise

$$\omega = \frac{2\pi}{\frac{2\pi r}{v}} = \frac{v}{r}$$

$$\boxed{v = r\omega}$$

$$a_c = \frac{v^2}{r} = \frac{(r\omega)^2}{r}$$

$$\boxed{a_c = r\omega^2}$$

Page 93

ex

Sol.

$$a_c = \frac{v^2}{r} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} \qquad \boxed{T = \frac{2\pi r}{v}}$$

$$a_c = \frac{4\pi^2 r}{T^2}$$

$$= \frac{4\pi^2 (1.496 \times 10^{11})}{(1\ yr} \left(\frac{1\ yr}{3.156 \times 10^7}\right)^2$$

$$= 5.93 \times 10^{-3}\ m/s^2$$

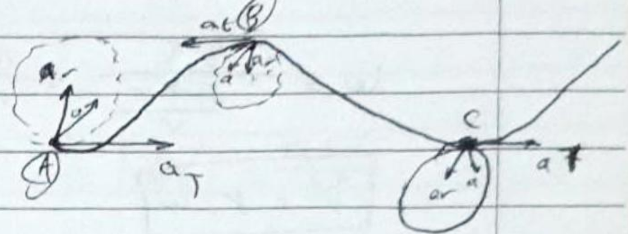
$$w = \frac{2\pi}{T}$$

$$w = \frac{2\pi}{1\,yr}\left(\frac{1\,yr}{3.156\times10^{7} s}\right)$$

$$w = 1.99\times10^{-7}\ s^{-1}$$

4.5 Tangential and radial acceleration

$$\vec{a} = \vec{a_r} + \vec{a_T}$$



$\vec{a_r}$ - radial comp. along radius of the Circle

$\vec{a_T}$ = tangential comp perpendicular to this radius

$$\boxed{\vec{a_t} = \frac{dv}{dt}}$$

$$\vec{a_r} = -a_c = \frac{-v^2}{r}$$

$$|\vec{a}| = \sqrt{a_r^2 + a_c^2}$$

if the speed is cons.

$a_t = 0$

$$\phi = \tan^{-1}\frac{a_r}{a_T}$$